21/11/2016

# Machine Design

- GATE [Brake]
  - approximately $\boxed{6\text{-}7 \text{ marks}}$
- ESE + Design of Flywheel
  - obj: $\boxed{9\text{-}10 \text{ qns}}$ $\boxed{16\text{-}17 \text{ marks}}$
  - Conventional: $\boxed{25\text{-}30 \text{ marks}}$

**ESE Design**

**Topics Removed** } not in Gate

① Belt & Rope Drives
② Coupling
③ Knuckle joint
④ Screw Jack

✓ Key → Gate + ESE
✓ Shaft

### * Engineering Design →

Need + Requirement → Process → Fullfill or complete or soln.

Process: Configuration (C), Material (M), Manufacturing Process (P)

Engineering Design is an iterative design making activity to convert Resources ~~[illegible]~~ optimally to satisfy the human need. The ultimate aim of design is to prepare a drawing or chart (that is to be a selection of appropriate shape; appropriate material; calculation for appropriate dimension and the selection of manuf. details), In such a way that the resultant m/c component should perform its functionality satisfactory (without any failure).

* step used while designing a m/c element →

**Step I** → specify the function of the machine element.

**Step II** → Define various load acting on the machine component while performing its functionality.

**Step III** → Selection of appropriate shape (all geometrical properties are known).

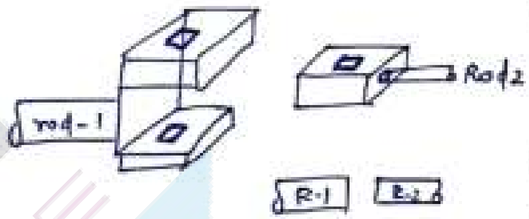
**Step IV** → Selection of appropriate material (all mechanical properties are known).

**Step V** → Define mode of failure (failure stresses are known under combined stresses condition).

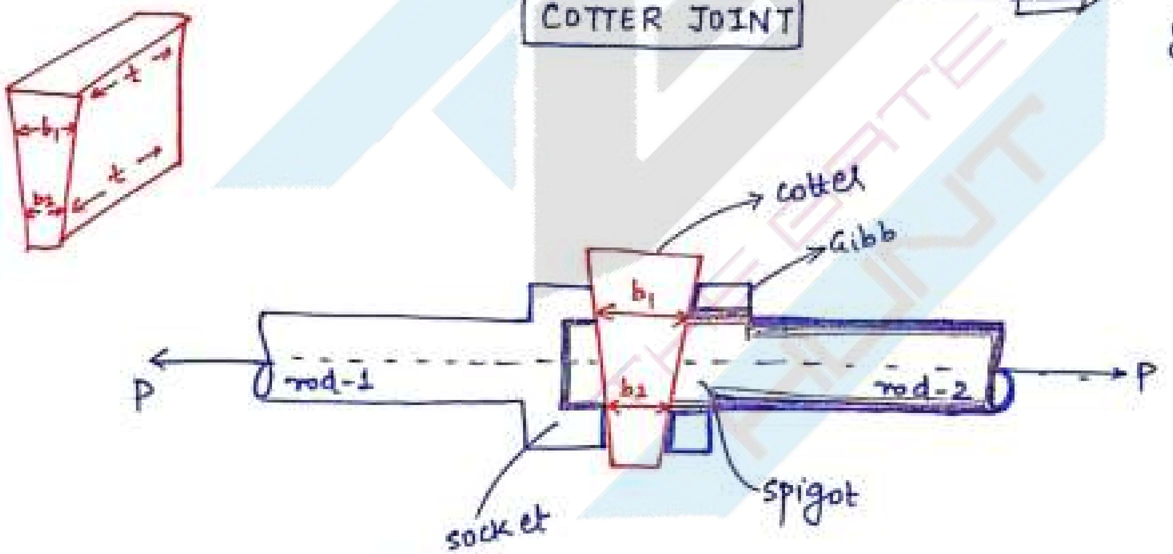
**Step VI** → Calc. of safe dimension by using SOM equation.

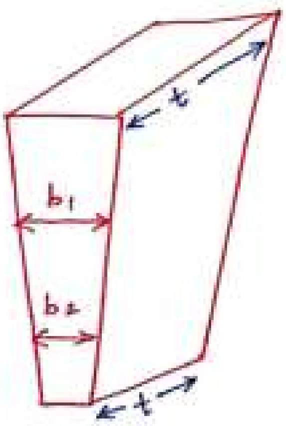
**Step VII** → selection of manufacturing details.

**Step VIII** → Prepare a drawing or chart.




## COTTER JOINT




[ spigot is used in aligning two Rods. ]

* cotter is a temporary fastener which is inserted b/w two co-axial Rods to transmit axial load from one Rod to another Rod.

Ex :- Train

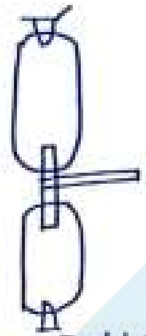
Coaxial means coaxial.

No use in Vehicle.

Train → पहले Before 2002, knuckle joint था)

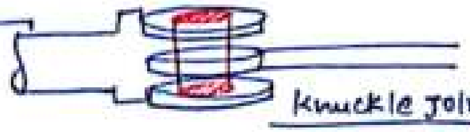
after 2002, we removed knuckle joint since speed problems, Now using CBC coupling

* Express trains



Centre Buffer Coupling.

coaxial as well as non-coaxial sft alignment



Knuckle Joint (Turning Pair)

Ex :- Tractor Trailer → modified version of cotter ↳ Knuckle joint.

Turning also

Knucke Joint → always Tension (Bear कर पाएगा)

But in cotter joint (Bears Tension as well as compression)

* old मालगाड़ी → knuckle joint can be seen

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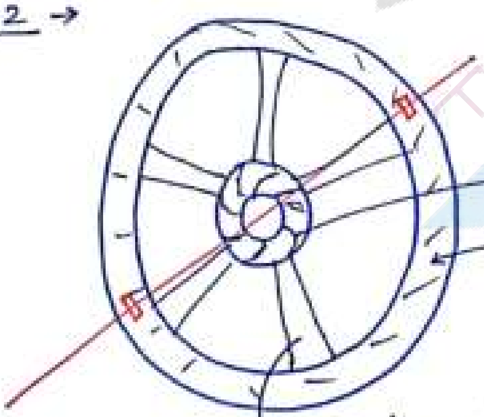
Cotter → Taper is provided into the width due to easy fastening purpose

Application of Cotter :- Example 1 → Connection between piston rod and crosshead.

Example 2 →




Connection b/w two hafs of flywheel
halfs

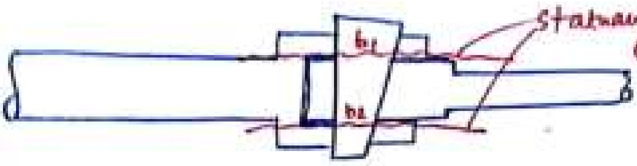
Example 3 → Foundation bolts.

Ex4 :- connection b/w Tail Rod and piston Rod.

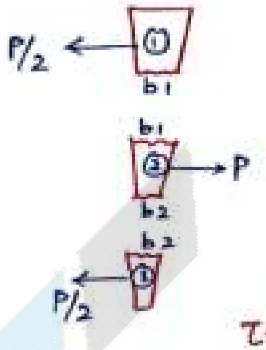
Cotter joints are weak in shear.

* ~~shear~~ shear Design of cotter :-

shear stress = $\frac{\text{shear load}}{\text{shear area}}$




$\tau_1 = \frac{P/2}{b_1 t} = \frac{P}{2b_1 t}$




$\tau_2 = \frac{P}{b_1 t + b_2 t} = \frac{P}{b_1 t + b_2 t}$

$\tau_3 = \frac{\frac{P}{2}}{b_2 t} = \frac{P}{2 b_2 t}$

$\tau_{max} =$

Safe condition

$\tau_{max} \le \tau_{per}$

$\frac{P}{2b_2 t} \le \tau_{per}$

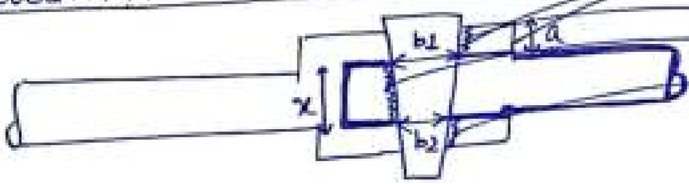
$P \le 2b_2 t \times \tau_{per}$

$\boxed{P_{max} = 2b_2 t * \tau_{per}}$

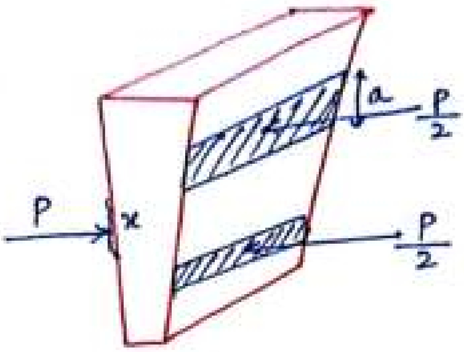
shear strength of cotter.

cotter completed.....

NOTE :-



## Crushing in cotter



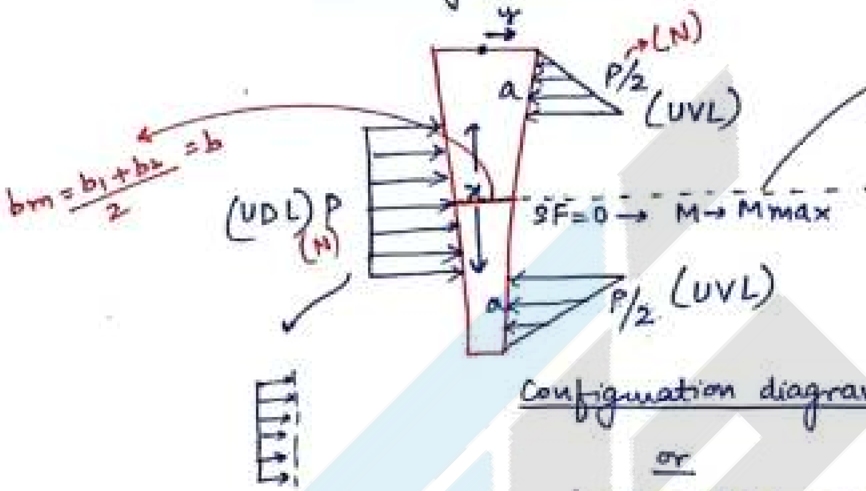

$$(\sigma_{ind})_{crush} = \frac{P}{2at}, \frac{P}{xt}$$

safe condⁿ:

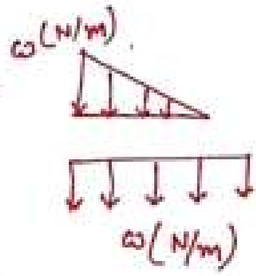
$$(\sigma_{ind})_{crush} \le \sigma_{perm}$$

$$\frac{P}{2at}, \frac{P}{xt} \le \sigma_{perm}$$

## Bending in cotter



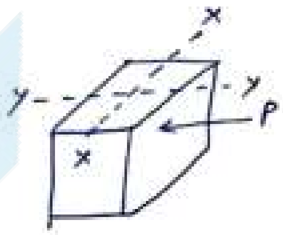

$$M_{max} = +\frac{P}{2}\left(\frac{a}{3}+\frac{x}{2}\right) - \frac{P}{2}\left(\frac{x}{4}\right)$$




Configuration diagram

or

Loading diagram




$$(\sigma_b)_{max} = \frac{M_{max} \cdot y_{max}}{I_{NA}}, \quad I_{NA} = \frac{bt^3}{12}, \frac{tb^3}{12}$$

$$y_{max} = b/2$$

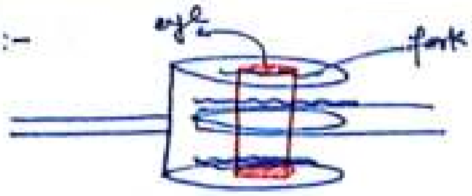
and $M_{max} = +\frac{P}{2}\left(\frac{a}{3}+\frac{x}{2}\right) - \frac{P}{2}\left(\frac{x}{4}\right)$

safe condⁿ → $\boxed{(\sigma_b)_{max} \le \sigma_{perm}}$

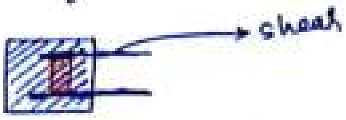
Now,
Weaker criteria can be considered mostly from the next time.

We can't Apply Theories of failures. Since No common point of Weaker point.

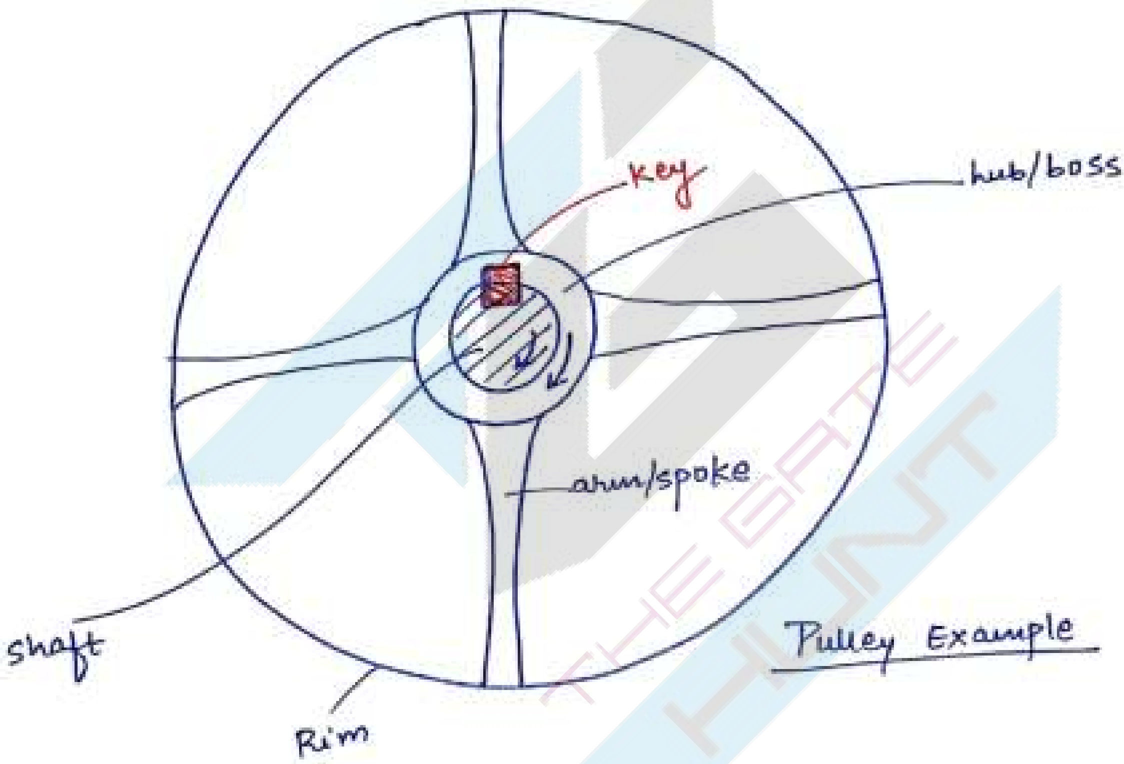
Hint :-



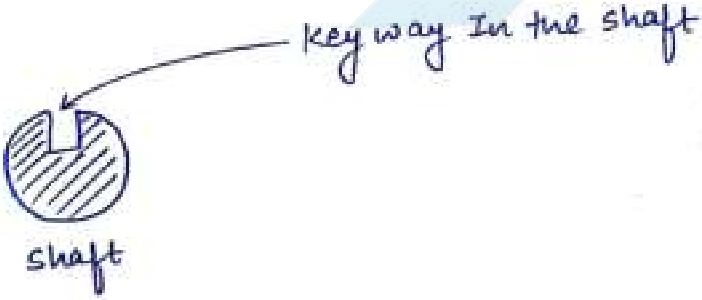

Top view of Gibb → crushing
→ shearing {not combined stress since alag alag jagah lagegi}

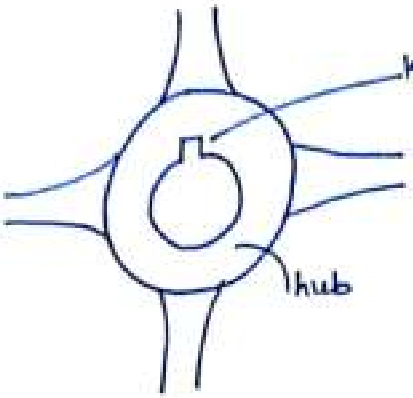


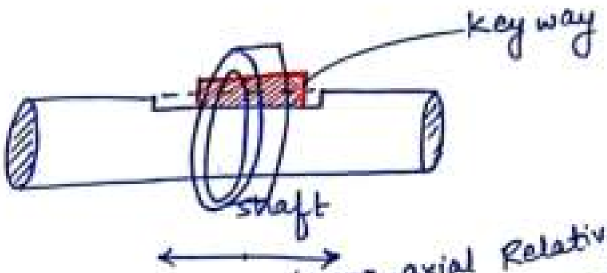

## KEYed JOINT




Pulley Example



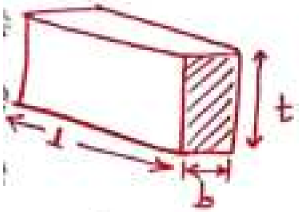



axial Relative motion so soldering करते key को]

Temporary है key is also key way

Ex:- Rivet is a temporary but after removing खराब हो जाता।




Key

**KEY:** key is a temporary fastener which is inserted b/w shaft and its assembly to transmit power by preventing Relative motion between them.

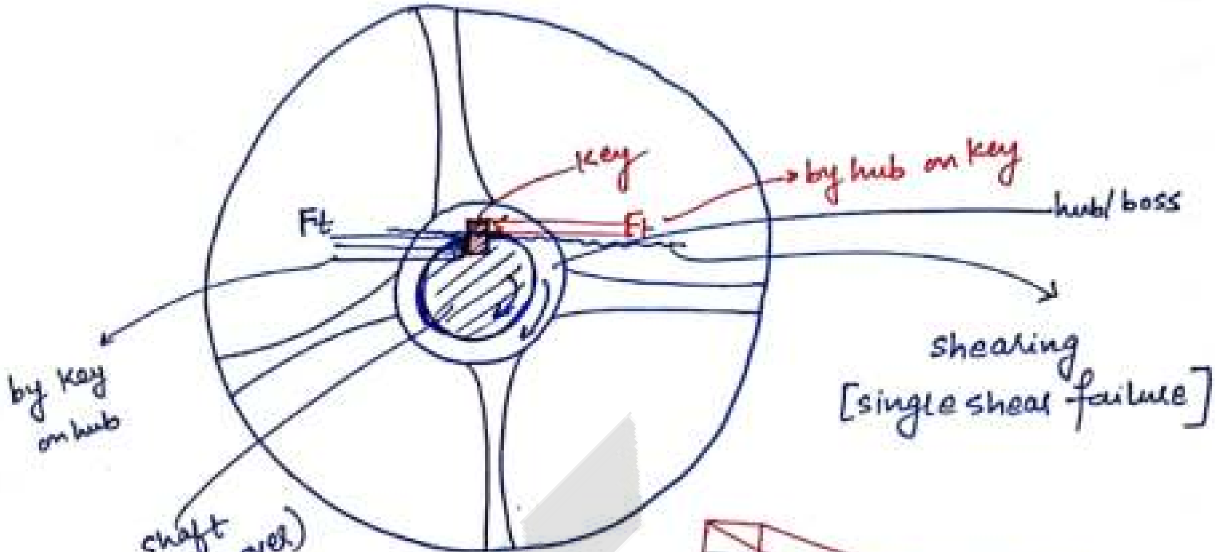
Soldering is done to avoid axial relative motion and to provide stability.

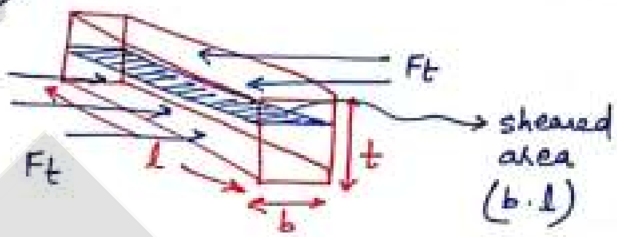
Key is the weakest element among shaft; assembly and key.

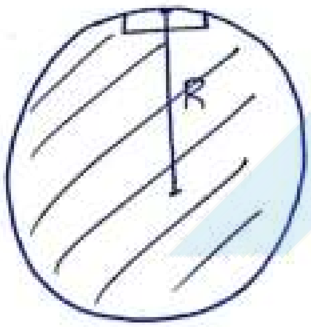
Assembly may be :-
(a) pulley,
(b) gear,
(c) flywheel,
(d) Clutch,
(e) Coupling, etc.

Key is weak in both shearing as well as crushing.

## Shear Design of the Key




$F_t \times R = T$

$\boxed{F_t = \frac{2T}{D}}$, $D \to$ dia of shaft

$\tau_{key} = \frac{F_t}{b \cdot l}$

can be convert into form of Torque. ($\because$ Rotation)

Now, $\tau_{key} = \frac{2T}{Dbl}$

safe condition,

$\tau_{key} \le \tau_{perm}$.

$\frac{2T}{Dbl} \le \tau_{per}$.

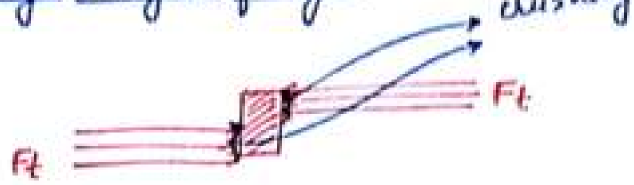
Generally

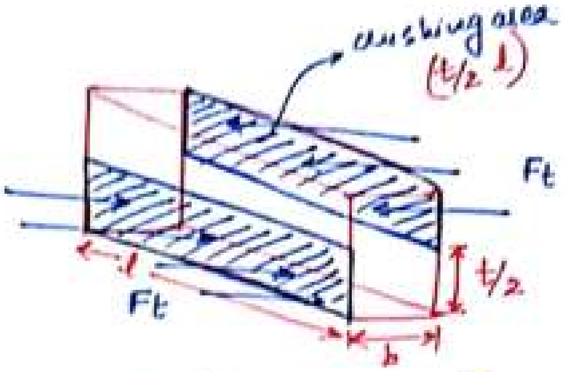
$b = D/4$

$\boxed{T_{max} = \frac{Dbl}{2} \tau_{per}}$

shear strength of key in terms of Torque

## Crushing Design of Key




### Safe Condition

$$(\sigma_{ind})_{crushing} \le \sigma_{per}$$

$$\frac{4T}{Dtl} \le \sigma_{per}$$

$$\boxed{T_{max} = \frac{D t . l . \sigma_{per}}{4}}$$

$$(\sigma_{ind})_{crushing} = \frac{F_t}{\frac{t}{2} . l} = \frac{4T}{Dtl}$$

crushing strength of key in terms of Torque

⇒ Actual strength of key = min. of $[(T_{max})_{shear}, (T_{max})_{crushing}]$

↳ max amount of Torque that key can transmit neither shear & crushing

Let $l_{shear} = 15mm$

$l_{crush} = 18mm$

$\left( \because \frac{4T}{Dt(l)} \le \sigma_{per} \right)$

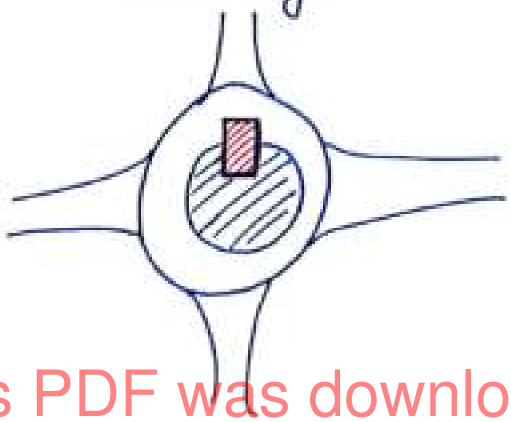
key is safe but not by crushing

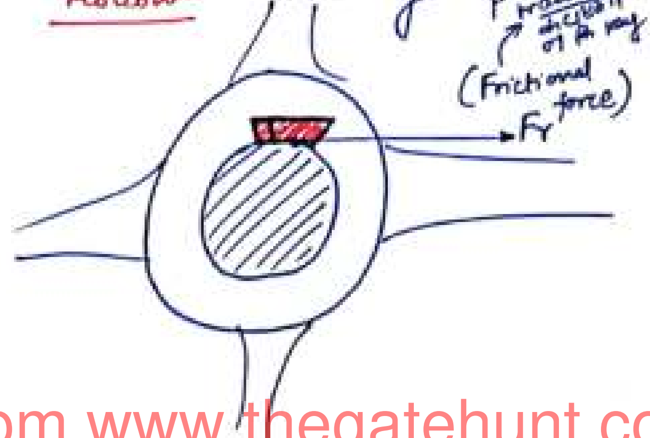
⇒ Actual Length of key = max. of $[l_{shear}, l_{crushing}]$.

# TYPE OF KEY :-

Type of key

- Sunk key
- Hollow - Saddle key





## Sunk key

1) half part of the key is in the hub and another half is in the shaft hence, keyway require in both hub as well as shaft.

2) key is responsible to transmit power.

3) used for high power transmission.

4) strength ↓, cost ↑.

## saddle key

① key is only in the hub not in the shaft, hence keyway req. only in the hub.

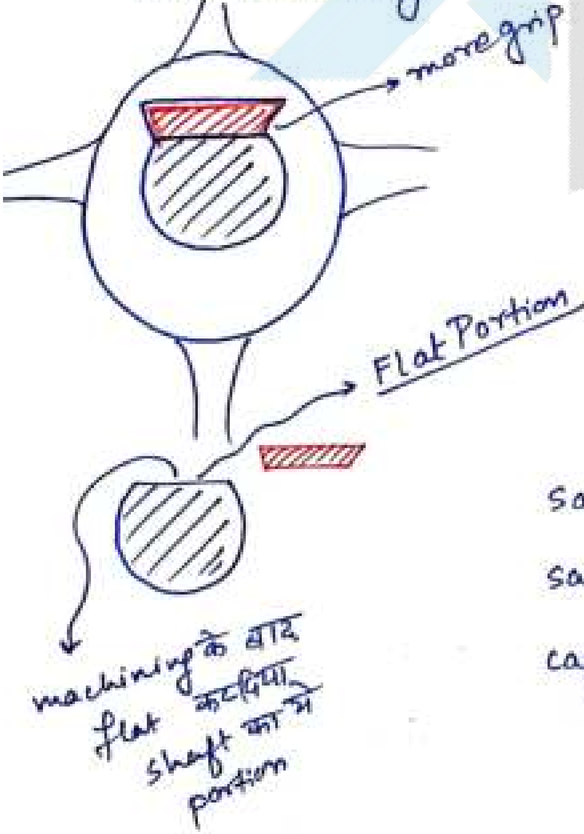
② friction force is responsible to transmit power.

③ used for low power transmission only.

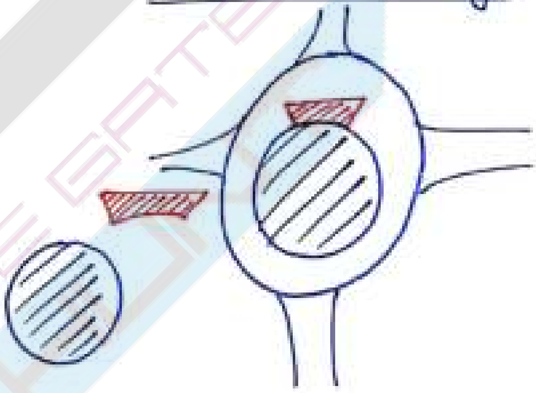
④ Because of no keyway present in the shaft, strength of the shaft increases (stress concn. factors are less) and cost decreases.

## Saddle key

Flat saddle key



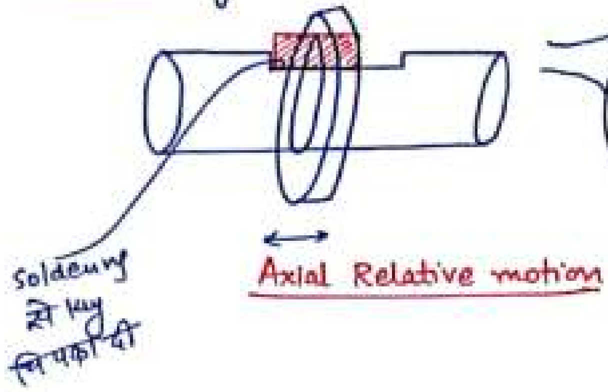

hollow-saddle key

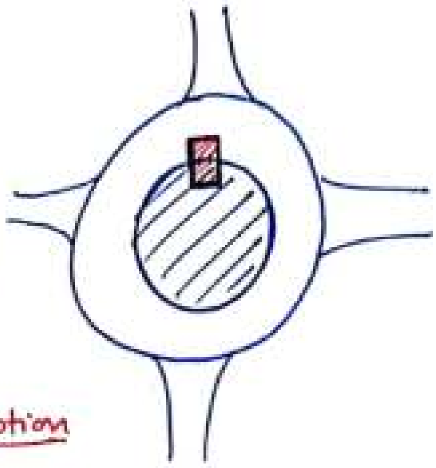


Flat saddle key is more superior than hollow saddle key wrt power transmission capacity.

# Type of Sunk key :-

① Feather key :-



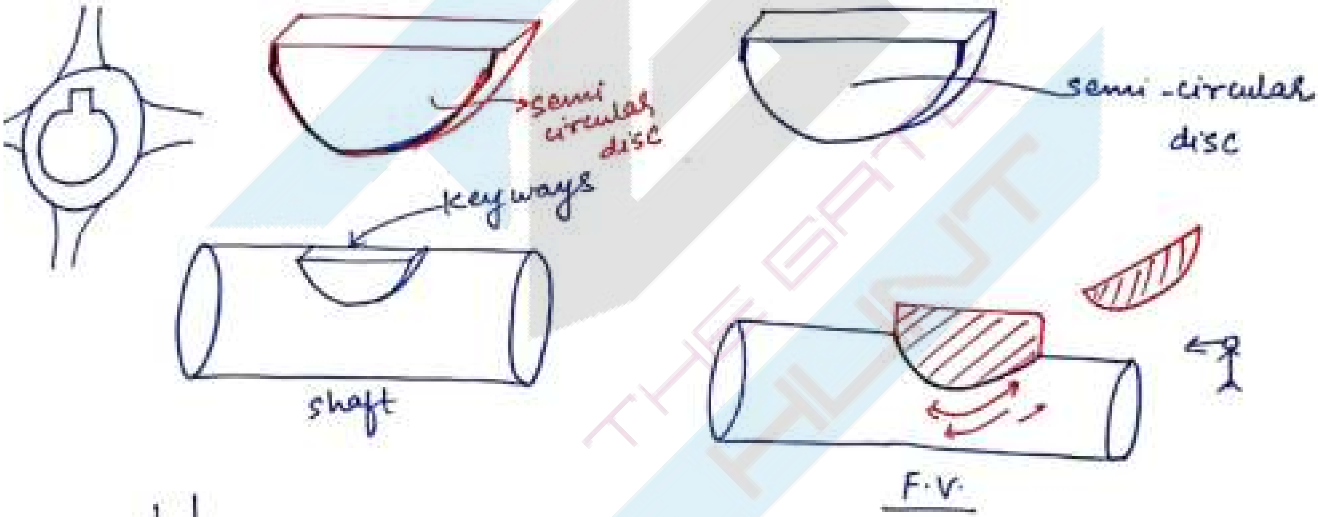



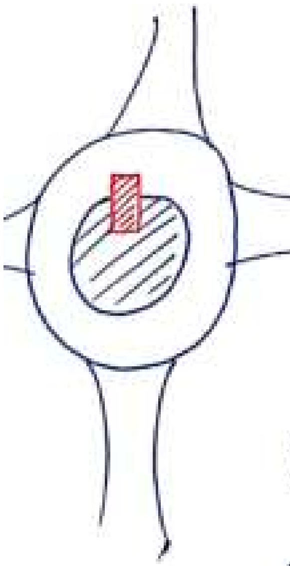
ⓐ key is fixed either with the shaft or with the hub.

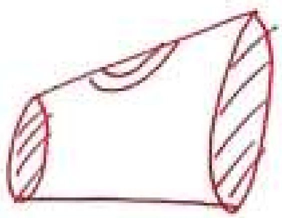
ⓑ permit axial Relative motion b/w shaft and its assembly.

ⓒ It is a type of parallel key.

② Wood Ruff key :-



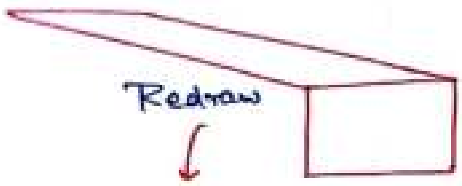


ⓐ key is looking like semicircular disc hence keyway is also semicircular disc.

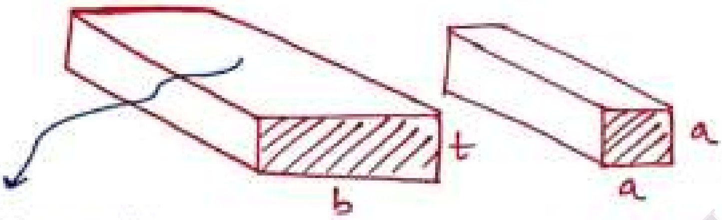
ⓑ It can align itself, hence can be used in Tapered shaft.

ⓒ extra depth of the key in the shaft provide more power transmission capacity.

## ③ Rectangular and Square sunk key :-




Rectangular sunk key is more stable than Square sunk key and used most of the industrial application

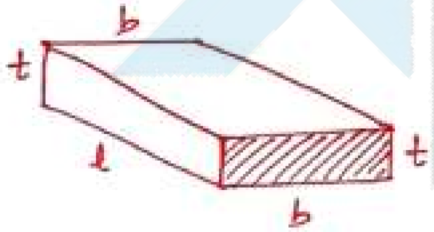


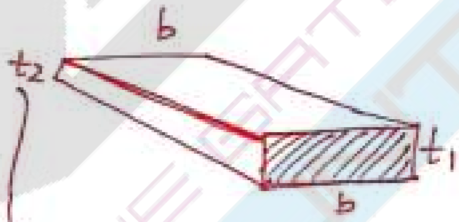

This area is more hence this key is more stable

Rectangular sunk key or flat sunk key

square sunk key

## ④ Parallel And Taper Sunk key :-




Rectangular Taper sunk key

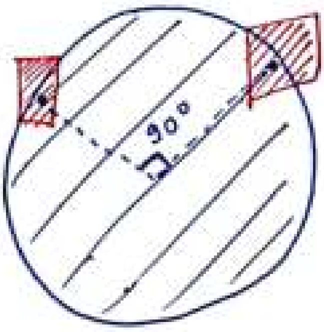
Parallel sunk key

Rectangular

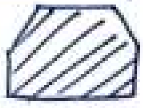
thickness is Tapered (cotter $\Rightarrow$ width is Tapered).

Taper is provided into the thickness due to Easy fastening purpose.

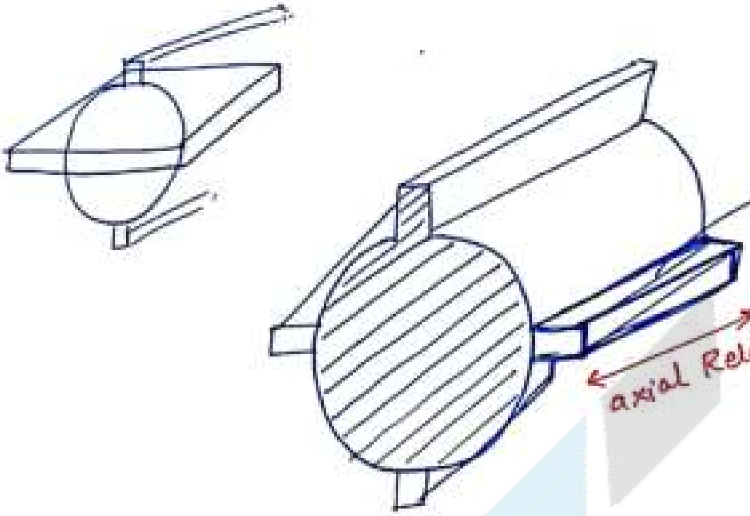
## ⑤ Kennedy key :-

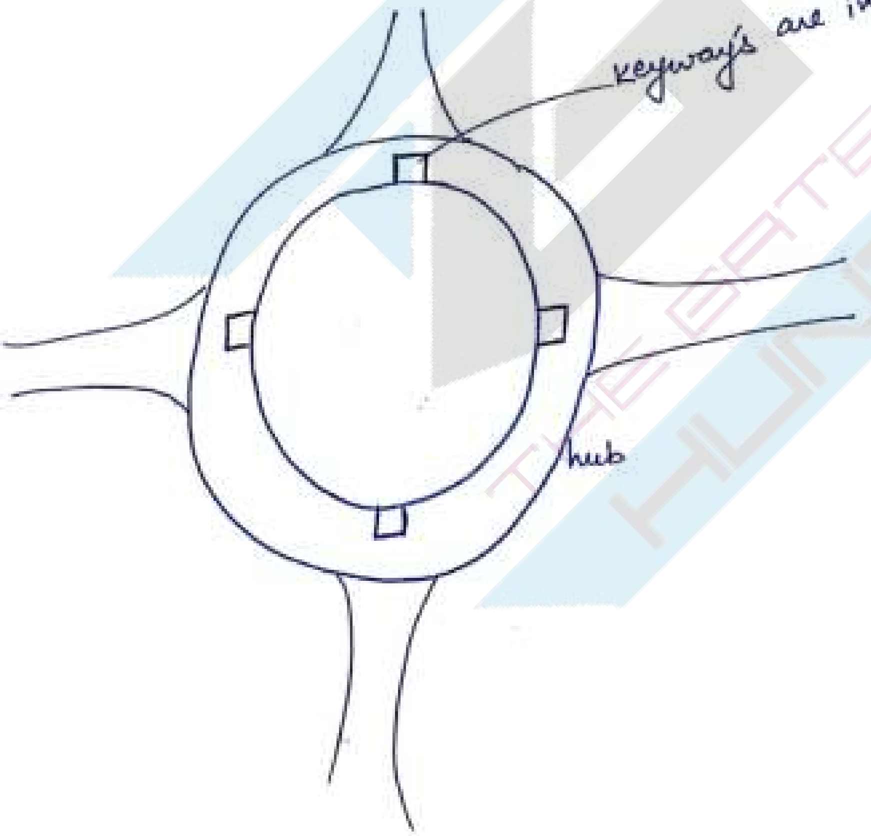


⑥ Barth key → more stable



** Splined Key or Splined shaft




→ To permit axial Relative motion b/w shaft and its assembly
→ used in automobile gear boxes & clutches.

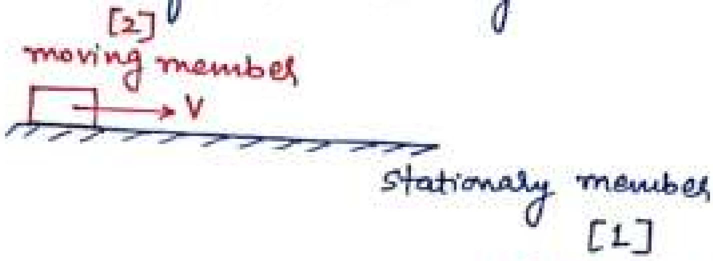
V.V. Imp

GATE/ESE

NEW CHAPTER

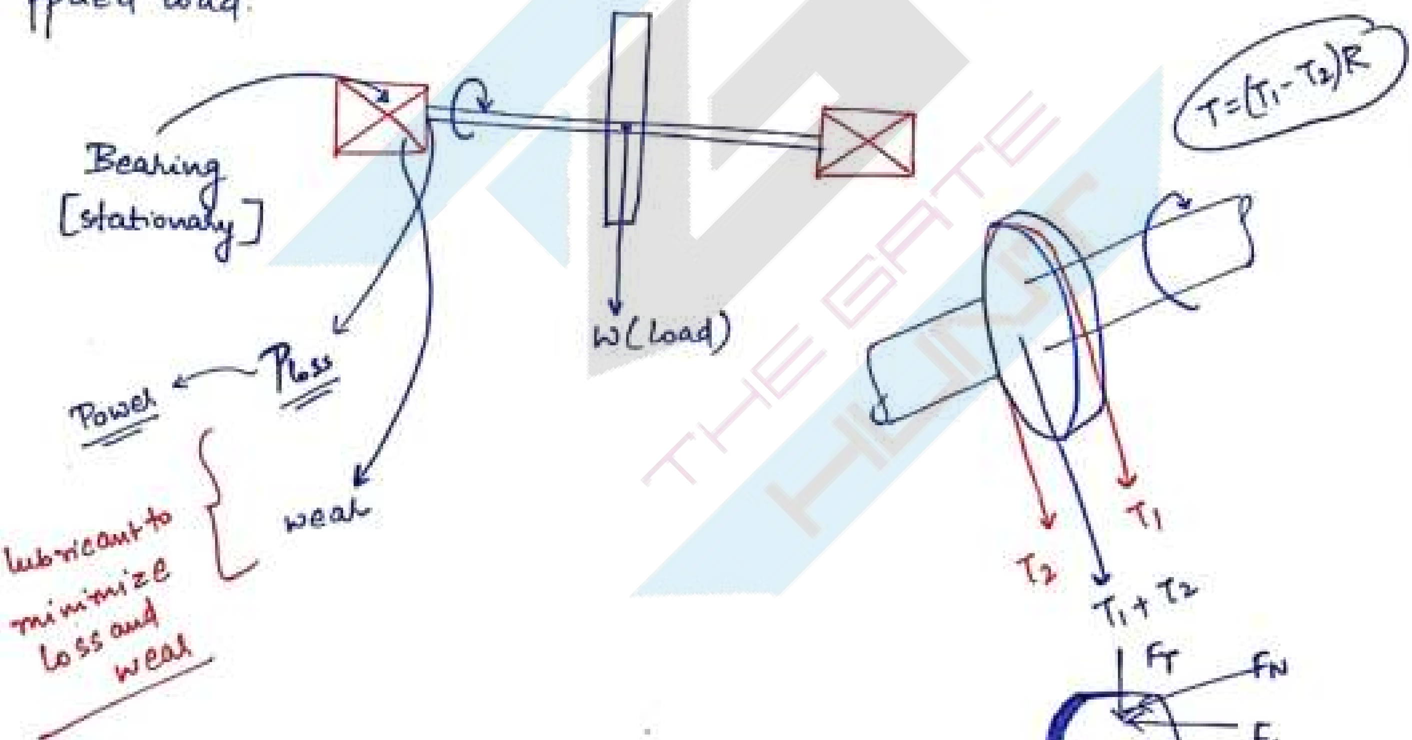
# BEARING

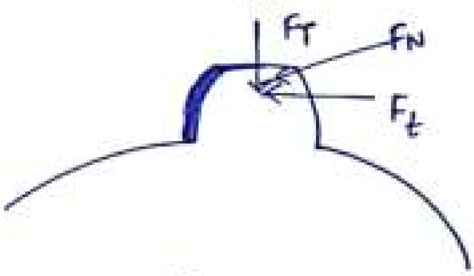
Whenever Relative motion occurs between 2 machine elements, the machine element which is stationary and supporting the moving m/c element is referred as Bearing.




[Bearing]

<u>Bearing According to the shaft</u> :- ① Bearing is defined as a m/c element whose function is to support a Rotating element (shaft or axle), to guide, to align and confined its motion while preventing the motion into a dirn. of applied load.




There is always frictional loss occurs while overcoming the frictional resistance and also wear occurs, hence lubricant required to minimise losses and wear.

A Bearing is said to be a good bearing which perform our fn. with minm. losses and wear.

Axle → No - Power transmission

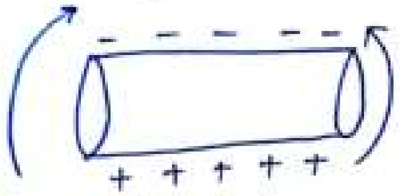
⇓

No Torque

⇓

No-Twisting

only Bending



axle are design by Bending only.

Shaft (Practical)

Aim → Power transmission

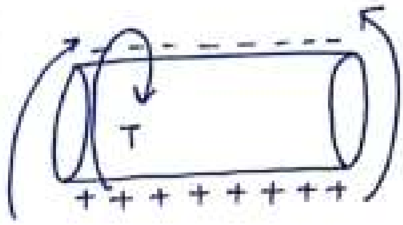
⇓

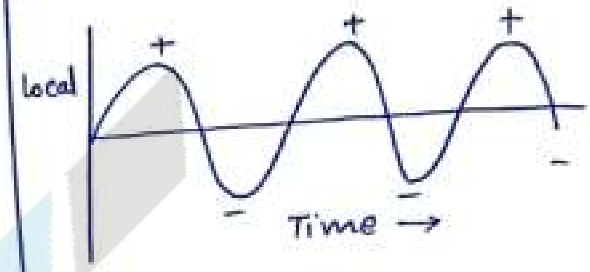
Torque

⇓

always in Twisting

Always Bending




⇒ Fatigue ⇒ Real shaft are Design by Fatigue.

# Classification of Bearing :-

# Bearing [stationary]

$\mu_{sliding} >>> \mu_{Rolling}$

- ~~sleeve Bearing~~ Sliding contact Bearing (sliding b/w two surfaces)
  - sleeve Bearing
    - w.r.t. dir$^n$. of load
      - Axial load Bearing or Thrust Bearing
        - Footstep Bearing / Pivot Bearing
          - Flat Pivot Bearing
          - conical Pivot Bearing
        - collar Bearing
          - Flat collar Bearing
          - conical collar Bearing
            - Single collar Bearing
            - Multicollar Bearing
      - Radial load Bearing or Journal Bearing
        - Full J.B.
        - Partial J.B.
        - Fitted J.B.
    - w.r.t. ~~dir~~ type of lubrication
      - Thin Film lubr$^n$. / Boundary lubrication
      - Thick film lubr$^n$
        - Hydrodynamic lubr$^n$
        - Hydrostatic lubr$^n$
      - zero Film or Solid lubr$^n$
    - wrt type of Bushes
      - Solid Bush Bearing
      - Split Bush Bearing
  - Slipper Bearing or Guide Bearing
- Rolling contact Bearing (Antifriction Bearing) (Rolling b/w two Surfaces)
  - Ball Bearing
    - Noiseless B.B. / Deep Groove B.B.
    - Angular contact B.B.
    - Self Aligning B.B.
    - Thrust B.B.
  - Roller Bearing
    - cylindrical R.B.
    - spherical R.B.
    - Taper R.B.
    - Needle R.B.

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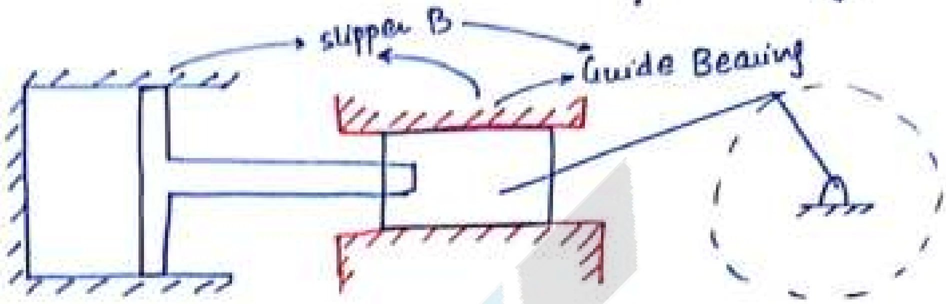
# Slipper or Guide Bearing → when sliding occurs in a straightline dirn., bearing referred as slipper Bearing. Hence, not used for the shaft.

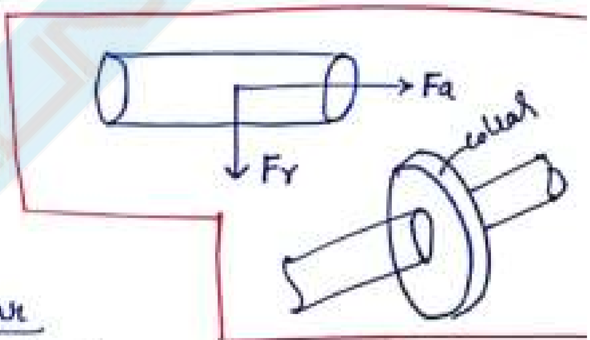

[Slipper Bearing]



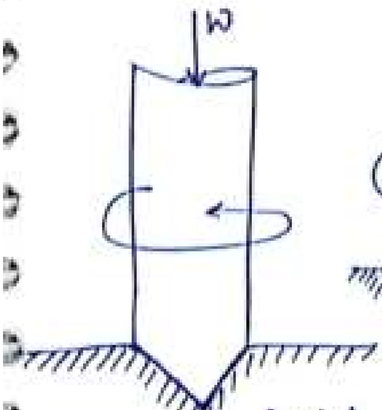



# Sleeve (B) → when sliding occurs in a circular direction ~~like~~ imply that around the periphery of the cylinder or circle, hence used for the shaft

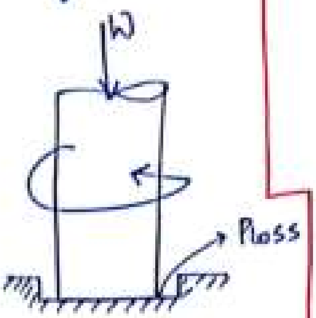
# Axial load Bearing :-
or
Thrust Bearing



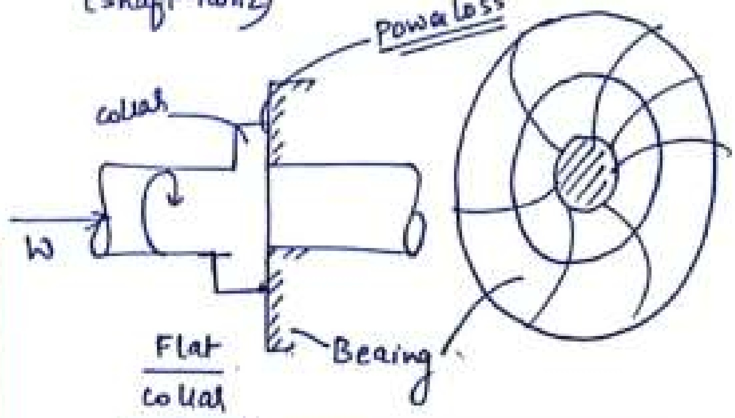

- Pivot (vertical shaft) — Footstep
- Collar (shaft-Horiz)



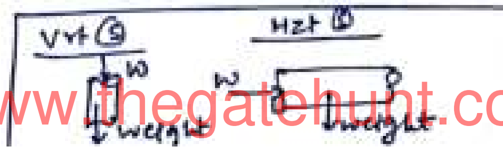

Conical Pivot

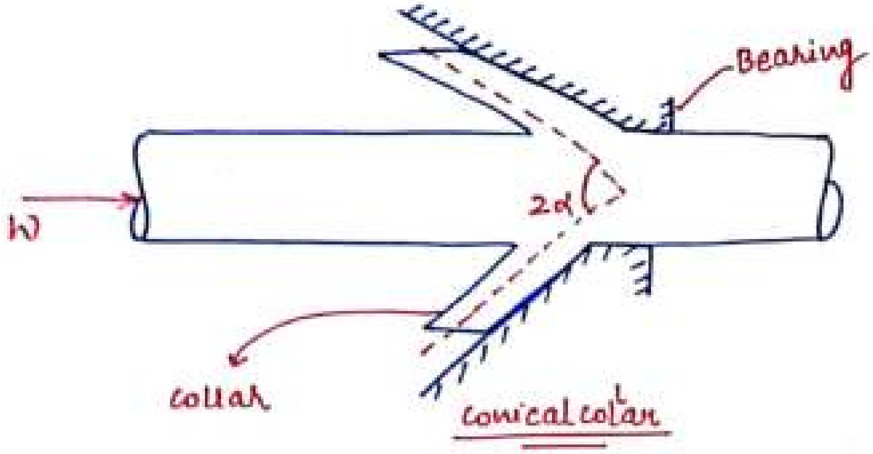



Flat Pivot




Flat collar

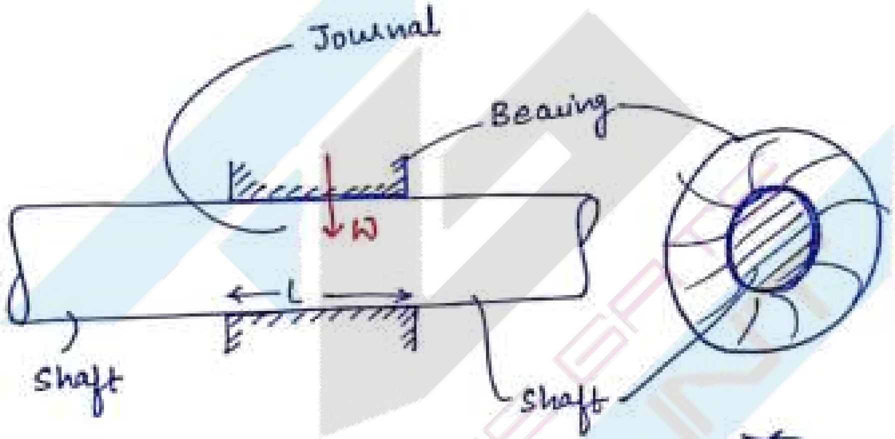


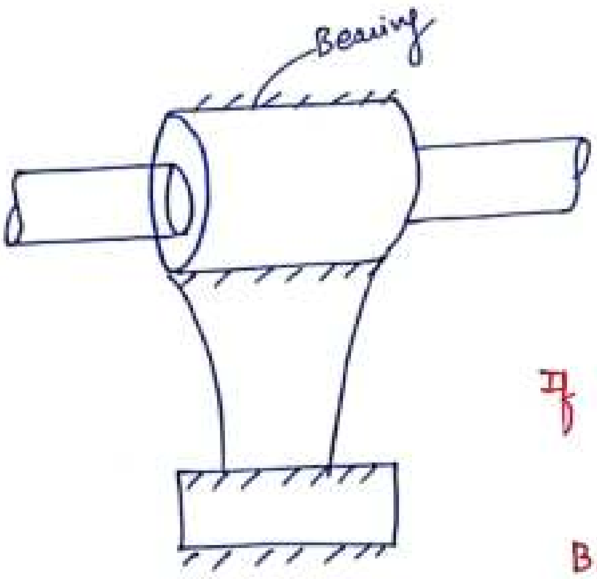



$2\alpha \rightarrow$ cone angle

$\alpha \rightarrow$ semi cone angle

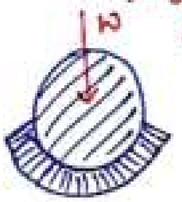
# Radial load Bearing →
or
Journal Bearing





Fitted - Full J.B.




→ force/load on only one dirⁿ.

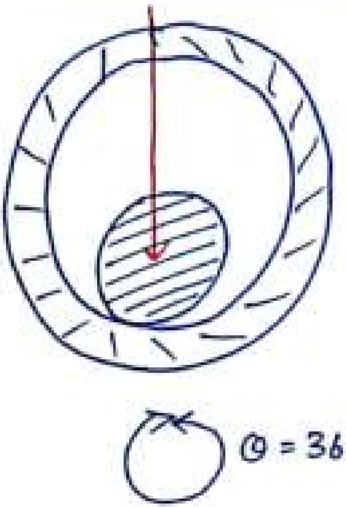
θ = 120°

Fitted - Partial J.B.

Ex:- [Rail Rod].

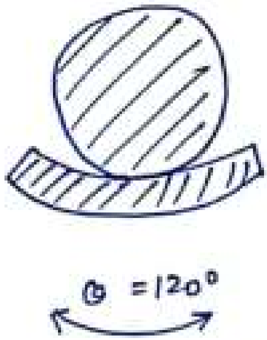
If Bearing dia. > shaft dia. ⇒ with clearance
(Inner)

Bearing dia. = shaft dia. ⇒ No clearance
⇓
Fitted



$\theta = 360°$

with clearance = Full J.B.



$\theta = 120°$

with clearance = Partial J.B.

. Partial J.B. can only be used when load is acting only in one direction.

# Types of lubrication → (wrt.)

① Solid Lubrication :- when metal behaves like a lubricant, referred as solid lubrication.

Ex:- ① Graphite . ③ Teflon.
② Cast Iron.

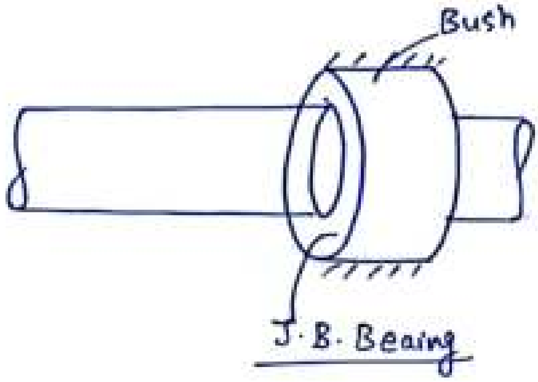
② Thin-film lubrication → metal to metal contact will present at any speed. The lubricant is used to reduce the coefficient of friction only.

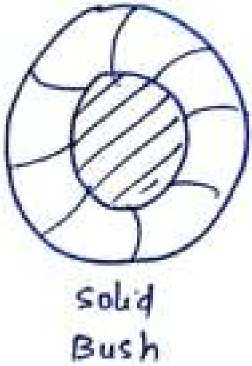
③ Thick-film lubrication → Metal to metal contact will always avoided.

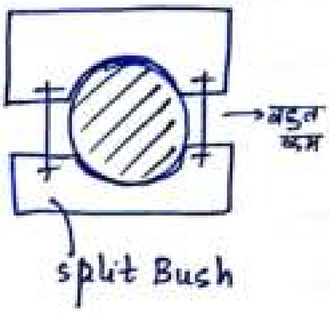
hydrostatic
metal to metal contact will/is avoided at stationary condition only

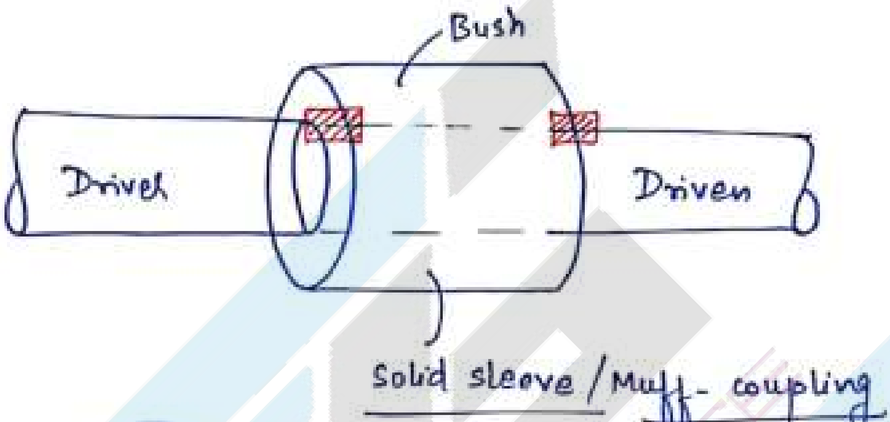
hydrodynamic
metal to metal contact is avoided only at high speed condn.

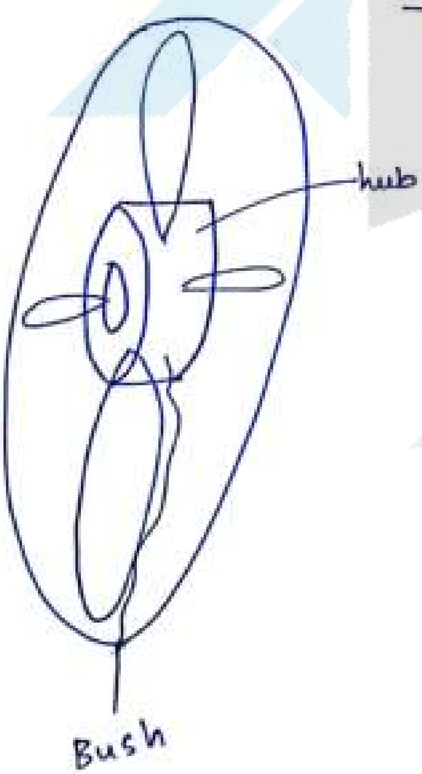
# wrt type of Bushes :-



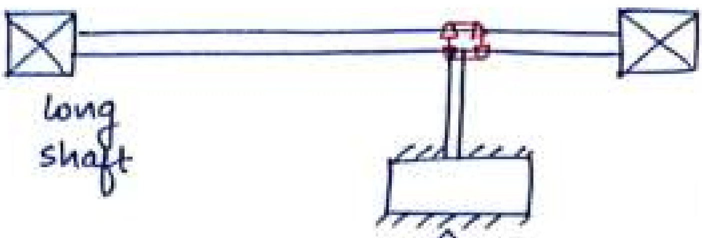




Defn. → Bush is a protector or cover, which protects the shaft all around its periphery.

In complicated situation, split Bush is used

$\delta = \frac{WL^3}{48EI}$ or $\frac{5}{384}\frac{WL^4}{EI}$

$L\uparrow \rightarrow \delta\uparrow$

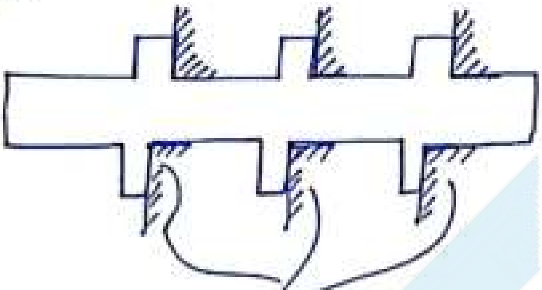


long shaft

Plummer Block

Plummer Block is used to provide intermediate support for long shaft

Type of split Bush ← Plummer Block → provides additional support to the shaft.



Split Bush

# Antifriction Bearing

AFB

Ball Bearing → Rolling element
↓
Balls
↓
point contact
↓
load capacity ↓
↓
loss ↓
↓
cost ↓

Roller Bearing → Rolling element
↓
Roller
↓
Linear contact
⇓
load capacity ↑
⇓
cost ↑
↓
loss ↑

# Ball Bearing

## Deep Groove B.B.

① $\frac{Fr}{Fa} > 1$

Fr → radial load

Fa → axial load

② Construction simple

③ cost ↓

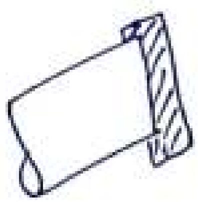
④ most commonly used.

⑤ Noiseless Ball Bearing

minm. noise in All antifriction Bearing (AFB).

Drawback :- common

• doesn't permit any misalignment b/w shaft and Bearing housing



## Angular contact B.B.

① $\frac{Fr}{Fa} < 1$

↳ axial load ज़्यादा Bear करती है।

② more stronger than D.G.B.B.

① Construction difficult (manufacturing difficult)

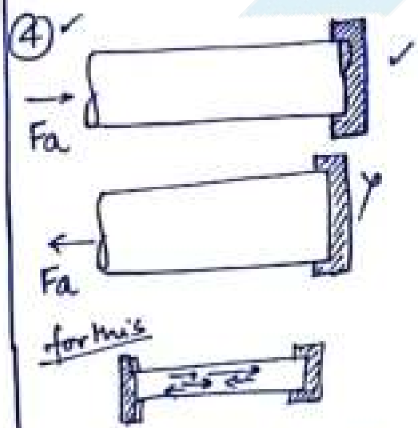
heavy load is req. by the balls to attach in the Race

constr. diff. means preloading of the bearing required.

② Preloading.

③ cost ↑.

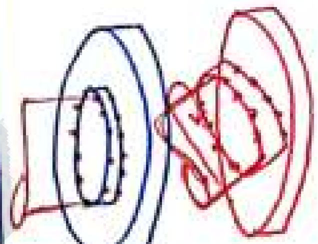
④ ✓




✓ Two Bearings are req. to bear axial load in both direction.

## Self Aligning B.B.

① Permit misalignment b/w the shaft and bearing housing.



– by Two Row of moving ball mechanism is used.

② bears also Fr & Fa.

## Thrust B.B.

① only bear Fa ✓ doesn't Fr = 0.

② They are prefer for vertical shaft (∵ weights are also in axial dirn.)

# Roller Bearing

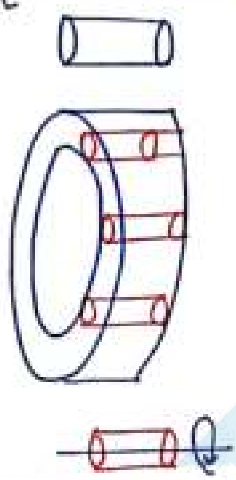
$L/D < 1$: Cylindrical R.B., Spherical R.B., Taper R.B.

Needle: $L/D > 1$

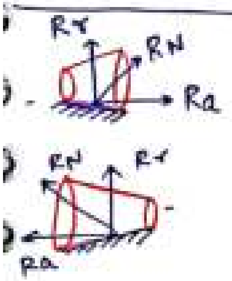
## Cylindrical R.B.

① $F_r$ ✓, $F_a = 0$

② more effective in case of [Hz shaft] {horizontal shaft}.



maxm. Radial load Bearing capacity

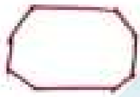



## Spherical R.B.

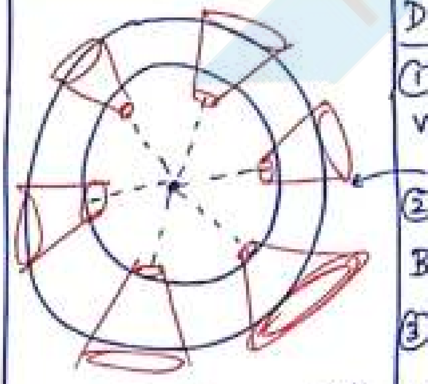
① $F_r$ ✓ $F_a$ ✓

② permit misalignment between shaft and Bearing housing.

③



\# 2 Row of moving Rollers are used.

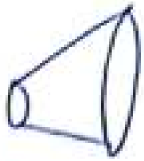


Tightening Nut

Ex:- Engine ~~of~~ shaft

## Taper R.B.

①



$\frac{F_r}{F_a} > 1$ ✓

Radial load Bearing capacity ज्यादा ज्यादा

② Maxm. load Bearing capacity

③ prefer under fatigue or impact loading.

Ex:- axles of Buses and Trucks, etc (ज्यादा load वाला)

Drawback→

① Construction is very difficult.

② preloading of the Bearing Required

③ Tightening nut in the Races required

④ Cost Maxm.

⑤ Taper Rollers always used in pairs.

⑥ does it permit any misalignment b/w ⓢ & ⓗ housing.

## Needle

① Needle Rollers are used when Radial space is a constraint. (कमी हो)

② maxm. load bearing capacity in a given Radial space.

③ $F_r$ ✓ $F_a$ ✓ can bear both loads.

Ex:- heavy loaded engines (piston), oscillating piston

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*

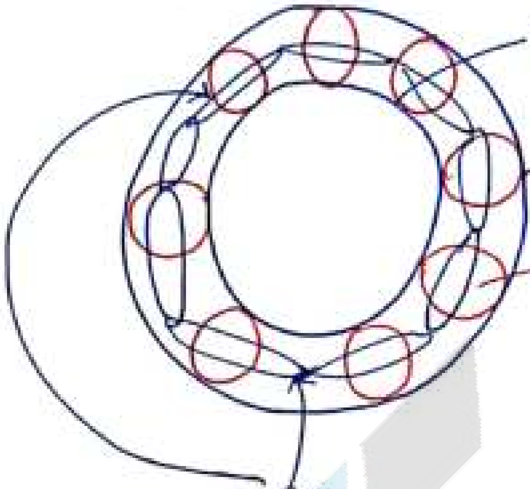
$\boxed{\text{Construction}}$

⇓

Inner Race
outer Race
Rolling element

} load Bear करें |

cage → सबको distance से बनाए रखना

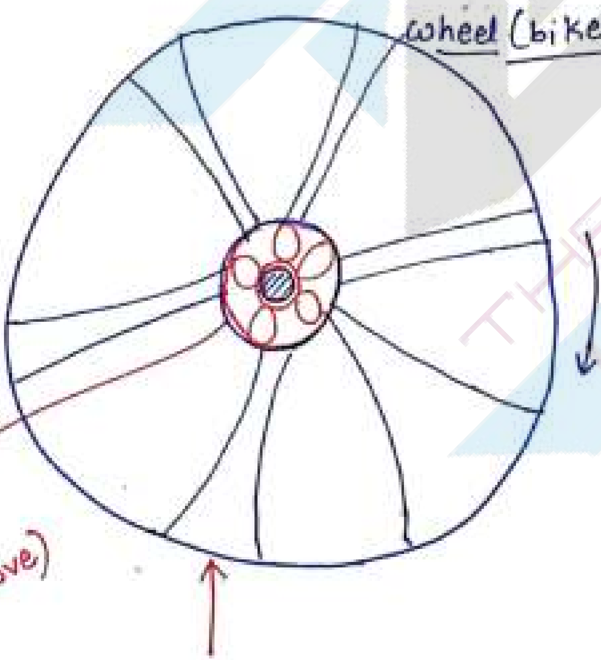



* **Function of Cage**

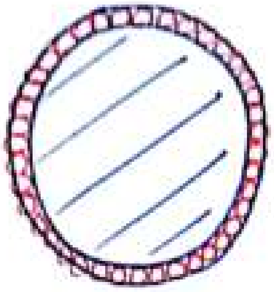
① Cage is used to maintain constant Relative angular position between two adjacent Rolling element.

② Cage is used to maintain the gap b/w Rolling element to avoid metal to metal contact to minimise losses and wear.

③ Cage is used to avoid clustering of the Rolling element at one location.




Note :- [ Cage is absent in case of needle Roller Bearing because needle rollers are placed around the periphery of the shaft.]



Needle Bearing

# FLAT COLLAR BEARING

$R_o$ = outer radius of collar

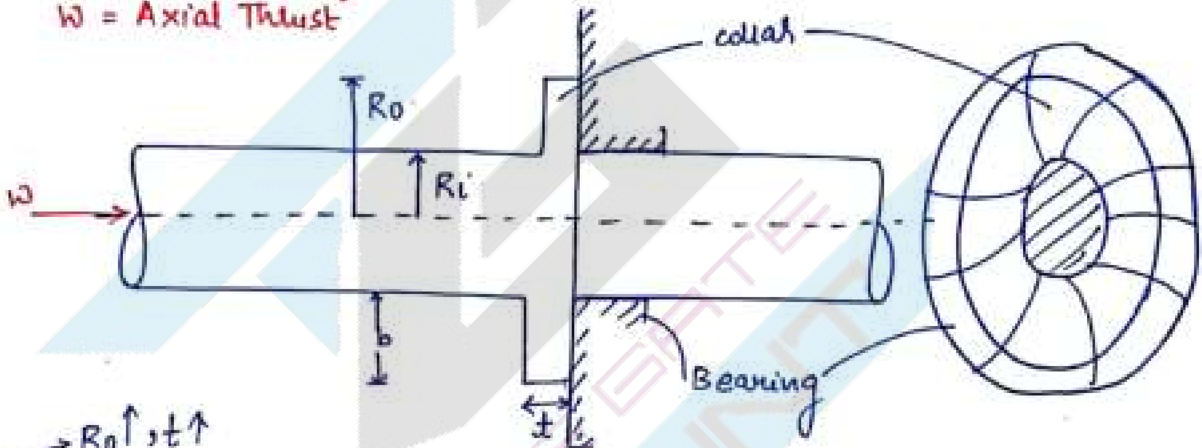
$R_i$ = inner ——— " ———

b = width of collar

$b = R_o - R_i$

t = Thickness of collar

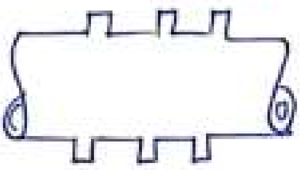
W = Axial Thrust




$W\uparrow$ → $R_o\uparrow, t\uparrow$

if Radial space is a constraint

→ $n\uparrow$ → no. of collar



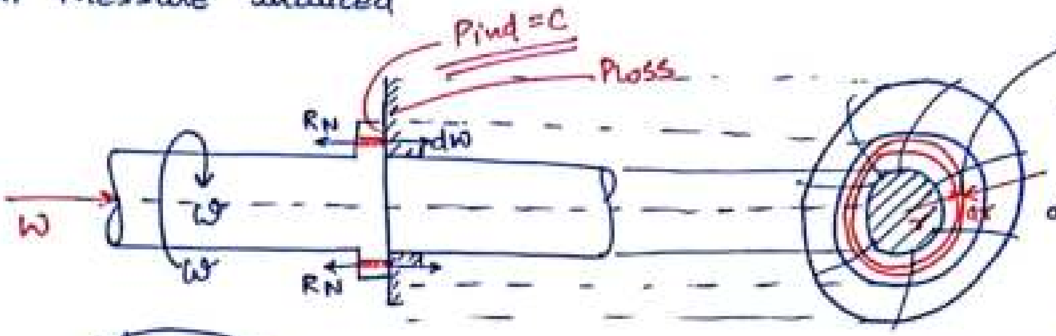
Flat collar Bearing

- single collar Bearing
- multicollar Bearing

✓ Single Collar Bearing :- (prev. Diagram)

(A) Design by Uniform Pressure Theory :- (UPT)

uniform Pressure induced

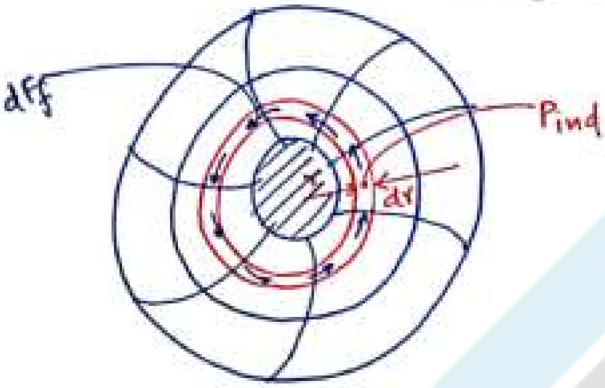



$P = C$ [UPT]

$$\int_0^W dW = \int_{R_i}^{R_o} (2\pi r dr) P$$

$$W = 2\pi P \int_{R_i}^{R_o} r dr$$

$$= \pi P (R_o^2 - R_i^2)$$




(1) $$\boxed{P_{ind} = \frac{W}{\pi(R_o^2 - R_i^2)}}$$ ✓

Safe Condition

$P_{ind} \le P_{per}$.

$$\frac{W}{\pi(R_o^2 - R_i^2)} \le P_{per}.$$

(2) $$\boxed{W_{max} = \pi(R_o^2 - R_i^2) P_{per}}$$ ✓

⇓

strength of collar by "UPT"

* Frictional Torque

$dF_f = \mu R_N$ → Rxn. $= dW$

differential frictional force.

$dF_f = \mu dW$

$dF_f = \mu \times 2\pi r dr \times P$

$dT_f = r dF_r$

$$\int_0^{T_f} dT_f = \int_{R_i}^{R_o} 2\pi\mu p r^2 dr$$

$$T_f = 2\pi\mu p \int_{R_i}^{R_o} r^2 dr$$

$$T_f = \frac{2}{3}\mu\pi p\,[R_o^3 - R_i^3]$$

$$P_{ind} = \frac{W}{\pi(R_o^2 - R_i^2)}$$

④ $$\boxed{T_f = \frac{2}{3}\mu W \frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}}$$

(UPT)

⑤ $P_{loss} = T_f \times \omega$ → angular velocity

## (B) Design by Uniform Wear Theory (UWT) →

(UWT)

Non-Uniform Pressure Induced

$$P_{ind} \propto \frac{1}{r} \Rightarrow P_{ind} = \frac{C}{r}$$

$$P_{ind} = P$$

$$P = \frac{C}{r} \quad (UWT)$$

$$\int_0^{W} dW = \int_{R_i}^{R_o} (2\pi r\, dr) P$$

$$W = 2\pi \int_{R_i}^{R_o} \cancel{r}\, dr \frac{C}{\cancel{r}} = 2\pi C \int_{R_i}^{R_o} dr$$

$$C = \frac{W}{2\pi(R_o - R_i)}$$

$$P = \frac{C}{r}$$

① $\boxed{P_{ind}\uparrow = \frac{W}{2\pi r\downarrow(R_o - R_i)}}$

<u>safe condn.</u>

$$(P_{ind})_{max} \le P_{per}$$

$$(P_{ind})_{max} = \frac{W}{2\pi R_i (R_o - R_i)}$$

$$\frac{W}{2\pi R_i (R_o - R_i)} \le P_{per}$$

② $\boxed{W_{max} = 2\pi R_i (R_o - R_i) P_{per}}$

strength of collar by UWT

$$dF_f = \mu R_N$$

$$dF_f = \mu dW$$

$$dF_f = \mu 2\pi r dr \cdot P$$

$$dT_f = r\, dF_r$$

$$\int_0^{T_f} dT_f = \int_{R_i}^{R_o} 2\pi \mu p\, r^2 dr$$

$$T_f = 2\pi\mu \int_{R_i}^{R_o} \cancel{r^2} \cdot \frac{c}{\cancel{r}} dr = 2\pi\mu c \int_{R_i}^{R_o} r\, dr$$

$$T_f = \mu \pi c (R_o^2 - R_i^2)$$

$$\frac{W}{2\pi c (R_o - R_i)} = c$$

③ $\boxed{\underset{UWT}{T_f} = \dfrac{\mu W (R_o + R_i)}{2}}$

④ $\boxed{P_{loss} = T_f \times \omega}$

Table

| UPT (uniform Pressure Theory) | UWT (uniform Wear Theory) |
|---|---|
| • UPT ⇒ $P_{ind} = C$ | • UWT ⇒ $P_{ind} \propto \frac{1}{r}$ |
| • $P_{ind} = \dfrac{W}{\pi (R_o^2 - R_i^2)}$ | • $P_{ind} = \dfrac{W}{2\pi r (R_o - R_i)}$ |
| • safe condn. $P_{ind} \le P_{per}$ | • safe cond. $(P_{ind})_{max} \le P_{per}$ |
| • $W_{max} = \pi (R_o^2 - R_i^2) P_{per}$ | • $W_{max} = 2\pi R_i (R_o - R_i) P_{per}$ |
| • $T_f = \frac{2}{3} \mu W \left( \dfrac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right)$ <br> $T_f = \mu W R_{eff}$ <br> $\boxed{R_{eff} = \frac{2}{3} \left( \dfrac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right)}$ | • $T_f = \dfrac{\mu W (R_o + R_i)}{2}$ <br> $\boxed{R_{eff} = \dfrac{R_o + R_i}{2}}$ |
| • $P_{loss} = T_f \cdot \omega$ | • $P_{loss} = T_f \cdot \omega$ |

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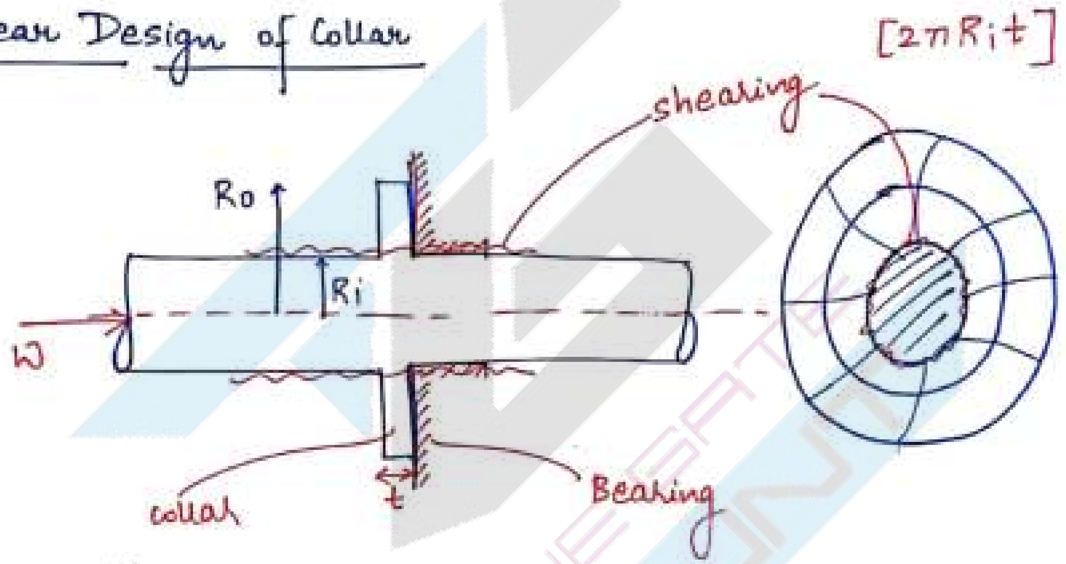
| (uniform press. Theory) UPT | (uniform wear UWT theory) |
| --- | --- |
| • $\tau_{ind} = \frac{W}{2\pi R_i t}$ | • $\tau_{ind} = \frac{W}{2\pi R_i t}$ |
| • $W_{max\ shear} = 2\pi R_i t \cdot \tau_{per}$ | • $W_{max\ shear} = 2\pi R_i t \cdot \tau_{per}$ |

# Shear Design of Collar




$$\tau_{ind} = \frac{W}{2\pi R_i t}$$

Shearing के लिए $t \uparrow$
crushing के लिए $R_o \uparrow$
→ dono ↑ for safety

safe condn.

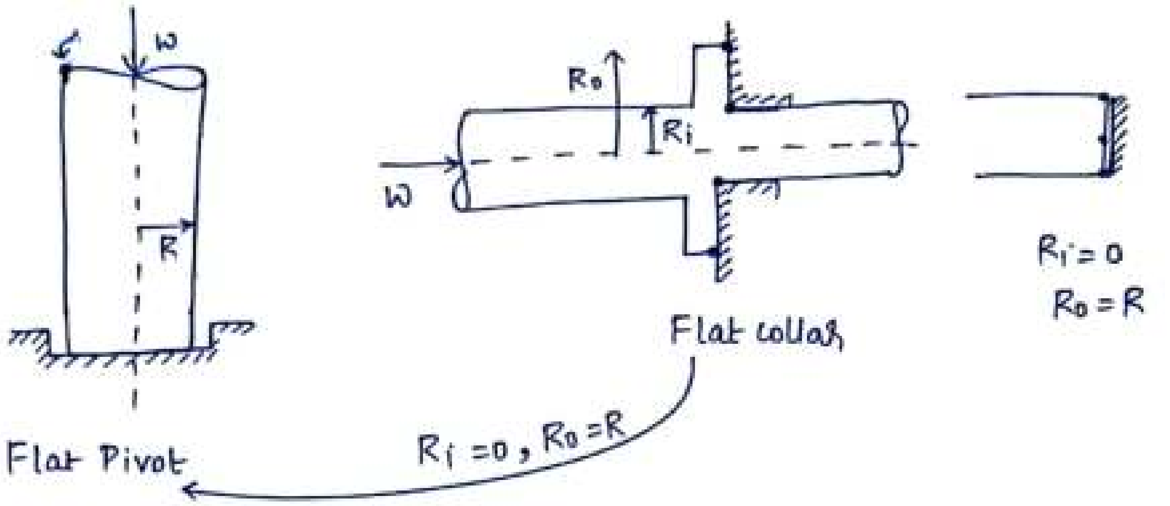
$$\tau_{ind} \le \tau_{per}$$

$$\frac{W}{2\pi R_i t} \le \tau_{per}$$

$$\boxed{W_{max} = 2\pi R_i t \cdot \tau_{per}}$$

⇓ shear strength of collar

# Expression for frictional Torque in case of Flat Pivot Bearing:-




Flat Pivot Bearing

① $\boxed{T_{f(UPT)} = \frac{2}{3}\mu W (R)}$ , $\boxed{R_{eff} = \frac{2}{3} R}$

② $\boxed{T_{f(UWT)} = \frac{\mu W R}{2}}$ , $\boxed{R_{eff} = \frac{R}{2}}$

③ $\boxed{\frac{T_{f(UPT)}}{T_{f(UWT)}} = \frac{4}{3} = 1.33}$

$T_{f(UPT)}$ ka load बड़ा है।

* Conclusions:- ① $T_{f(UPT)}$ is 33% greater than frictional Torque by UWT in case of flat pivot bearing.

② There is always frictional torque by UPT is greater than frict Torque by UWT. This is true for all cases.

③ $\boxed{(T_f)_{UPT} > (T_f)_{UWT} \quad (\text{for all cases})}$

$(T_f)_{UPT} > (T_f)_{UWT}$ (for all cases)

Bearing
↓
Aim
↓
support 'W'
↓
collar
↓
$T_f$ → Power loss

Clutch
Aim → Power transmission
↓
friction lining
↓
$T_f$ → power Transmit

Design Engg → acc. to worst condn., design can be done.

Conclusion :→ ① for the safe design of Bearing; it is better to use uniform pressure theory because power loss occurs in overcoming the frictional resistance.

② for the safe design of clutches; it is better to use uniform wear theory because pressure is non-uniformly distributed over the clutch surfaces (clutch → means old clutch or worn out clutch) {केवल}

③ for the safe design of new clutches, it is better to use uniform pressure theory because pressure is uniformly distributed over the clutch surfaces when they are new.

WORKBOOK

Pg 82 Q no. 12

$R_o = \frac{100mm}{2}$, $R_i = \frac{40mm}{2}$ $Pa \to \frac{N}{m^2}$

UPT

$P = 2MPa$, $\mu = 0.4$ $T_f$

$$T_{f\,UPT} = \frac{2}{3}\mu W\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$$

$$= \frac{2}{3} \times 0.4 \times 2 \times 10^6 W \left(\frac{0.5^3 - 0.2^3}{0.5^2 - 0.2^2}\right)$$

$$\frac{W}{R_o^2 - R_i^2} = P\pi$$

$$T_f = \frac{2}{3}\mu P\pi\left(R_o^3 - R_i^3\right)$$

$$= \frac{2}{3} \times (0.4) \times 2 \times 10^6 \times \pi\left(.05^3 - .02^3\right) = 196\,N\text{-}m$$

Q6 $P = 5 \times 1000 W$

$N = 2000\,rpm$

$\mu = 0.25$

$R_i = 25mm$

UPT $P = 1\,MPa$

$$\frac{2\pi NT}{60} = 5 \times 1000$$

$$T = 23.88\,N\text{-}m$$

$$T_{f\,UPT} = \frac{2}{3}\mu P\pi\left(R_o^3 - R_i^3\right)$$

$$\frac{23.88}{2 \times 0.25 \times 1 \times 10^6 \times 3.14} = R_o^3 - R_i^3$$

$T_f$ ✓ $R_o = 39.4mm$

Q A single plate clutch has a $\mu = 0.3$ is designed. The outer diameter of the friction lining is 200 mm and inner diameter is 100mm. The intensity of pressure cannot exceed 1.5 MPa (permissible) by assuming UWT, the max^m. Torque that can be transmitted.

Sol $R_o = 100mm$ $P_{per} = 1.5MPa$

$R_i = 50mm$

$$T_f = \frac{\mu W (R_o + R_i)}{2}$$

$$T_f = 0.3 \frac{P \times 2\pi r (R_o - R_i)(R_o + R_i)}{}$$

$$\underline{T_f = 0.3 \times} \; 1.5 \times 10^6$$

SIR

UWT

$$T_f \uparrow_{max} = \mu W \uparrow_{max} \frac{(R_o + R_i)}{2}$$

$$W_{max} = 2\pi R_i (R_o - R_i) P_{per}$$

$$T_f = \mu \pi P_{per} R_i (R_o^2 - R_i^2)$$

$$= 0.3 (\pi)(1.5 \times 10^6)(.05)(.1^2 - .05^2)$$

$$\underline{T_f = 530.1 \text{ N-m}}$$

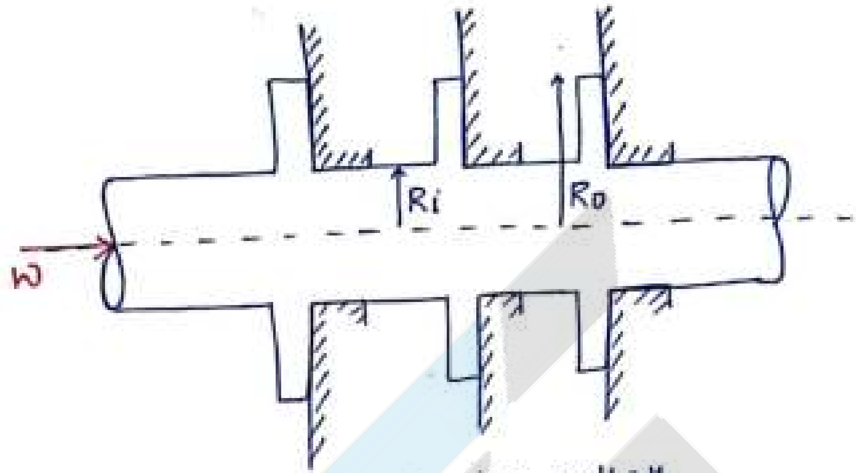
23/11/2016

## * MULTI COLLAR BEARING

$W \uparrow$ if radial space is a constraint

$n \uparrow$ = no. of collar

Bearing $\rightarrow$ UPT (uniform pressure theory)




similar collar

Safe condition

$P_{ind} \le P_{per}$

$$\frac{W}{n\pi(R_o^2 - R_i^2)} \le P_{per}$$

$$\boxed{W_{max} = n\pi(R_o^2 - R_i^2)P_{per}}$$

$\Downarrow$

strength of multicollar Bearing

$W_{collar}$ = load on each collar

$$W_{collar} = \frac{W}{n}$$

$$P_{ind} = \frac{W_{collar}}{\pi(R_o^2 - R_i^2)}$$

$$\boxed{P_{ind} = \frac{W}{n\pi(R_o^2 - R_i^2)}}$$



### * Frictional Torque and Power losses :-

$$T_f = n \cdot T_{collar}$$

$$T_f = n \left[ \frac{2}{3} \mu W_{collar} \frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} \right]$$

$$\boxed{T_f = \frac{2}{3} \mu W \left[ \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]}$$

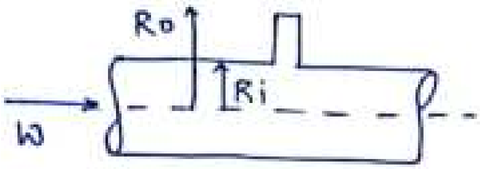
frictional torque is independent from no. of collar.

$$(T_f)_{\text{single collar Bearing}} = (T_f)_{\text{Multicollar Bearing}}$$

$$P_{loss} = T_f \cdot \vec{\omega}$$

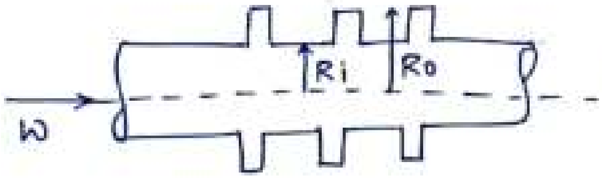
$$(P_{loss})_{S.C.B.} = (P_{loss})_{M.C.B.}$$

*



$W_{collar} = W$, $P_{ind} = P$, $T_f = T$, $P_{loss} = P'$

$\tau_{ind} = \tau / \boxed{\tau}$



$W_{collar} = W/3$, $P_{ind} = P/3$, $T_f = T$, $P_{loss} = P'$

$\tau_{ind} = \tau/3$

<u>collars</u> must be <u>identical</u>.

<u>Q</u> Which of the following statements are valid for multicollar thrust Bearing :-

① frictional movement is independent from no. of collars.

② coefficient of friction is affected by no. of collars.

③ Intensity of pressure is affected by no. of collars.

<u>①&③</u> ✓

<u>Q</u> (10 marks) The thrust of a propeller shaft in a marine engine is taken up by no. of collars inbuilt with the shaft which is 30cm in diameter. The axial thrust is 200 kN and speed is 75 Rpm, the coefficient of friction b/w surfaces is 0.05 and uniform intensity of pressure 0.3 MPa. Find out the external diameter of the collars and no. of collars required if power loss cannot exceed 16 kW.

<u>Sol</u> $D = 0.30\,m$

$W = 200\,kN$

$N = 75\,Rpm$

$\mu = 0.05$

$P = 0.3\,MPa$

$$0.3 \times 10^6 = \frac{200 \times 10^3}{n\pi(R_o^2 - R_i^2)}$$

$$P = \frac{2\pi N T}{60}$$

$$T_f = n \frac{2}{3} \mu W_{collar} \left( \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right)$$

$0.21 = n(R_o^2 - R_i^2)$

$W_{coll} = \frac{W_{collar}}{n}$ $\quad T_f = \frac{2}{3} \mu W \left( \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right)$

$P_{loss} = T_f \cdot \omega$

$0.08 = n \frac{2}{3} \mu W_{collar}$

S/R $P_{loss} = 16\,kW$ $\omega \rightarrow$ angular velocity

$P_{loss} = \frac{2\pi N}{60} T_f$ not load

$$16\times10^3 = \frac{2\pi(75)T_f}{60}$$

$$\boxed{T_f = 2037.18\ N\text{-}m}$$

$\neq$ P.T but = P.T.n — no. of collars can be multiplied.

$$T_f = \frac{2}{3}\mu W\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$$ → since multicollar question

$$2037.18 = \frac{2}{3}\times(.05)\times(200\times10^3)\times\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$$

$0.15^3$, $0.15^3$

$R_o = 24.92\,cm$

$D_o = 49.84\,cm$

$$P_{ind} = \frac{W}{n\pi(R_o^2 - R_i^2)}$$

$$.3\times10^6 = \frac{200\times10^3}{2\pi(.2492^2 - .15^2)}$$

$n = 5.37$

$\boxed{n = 6}$ ✓

Q. Design a multicollar thrust bearing to take an axial thrust of 16.3 Ton and the intensity of pressure cannot exceed 0.7 MPa; the outer dia. of the collars are 42 cm and inner diameter 32 cm and the safe shear stress for the collar material 25 MPa.

Sol $W = 16.3\times10^3\,Kg$

$P = 0.7\,MPa$

$D_o = 0.42\,m$ $R_o = 0.21\,m$ $P_{per} = 25\,MPa$

$D_i = 0.32\,m$ $R_i = 0.16\,m$

$$W_{max} = n\pi(R_o^2 - R_i^2)P_{per}$$

$$P_{ind} = \frac{W}{n\pi(R_o^2 - R_i)}$$

$$0.7\times10^6 = \frac{16.3\times10^3}{n\pi(0.21^2 - 0.16^2)}$$

$$7.41\times10^{-3} = (0.21^2 - 0.16^2)n$$

SIR. $R_o, R_i, b, t, n = ?$ $R_o = 21cm$ $R_i = 16cm$ $W = 16.3\times10^3 \times 9.81N$

$\boxed{b, t, n = ?}$

$b = R_o - R_i$

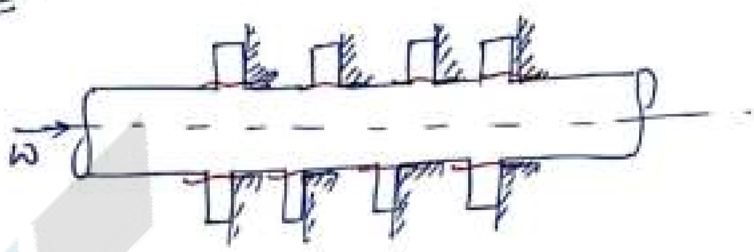
$\boxed{b = 5cm}$

$t = ?$

(a) 1.6 mm (b) 6.37mm (c) 5.6mm (d) 4.36mm

$t, n = ?$

collar (each)



$$P_{ind} = \frac{W}{n\pi(R_o^2 - R_i^2)}$$

$$.7\times10^6 = \frac{16.3\times10^3 \times 9.81}{n\pi(.21^2 - .16^2)}$$

$n = 3.9 \approx 4$

$\boxed{n = 4}$

$$\tau_{ind} = \frac{W_{collar}}{2\pi R_i t} = \frac{W}{4\times2\pi\times R_i t}$$

safe condn.

$$\frac{W}{4\times2\pi\times R_i t} \le 25\times10^6$$

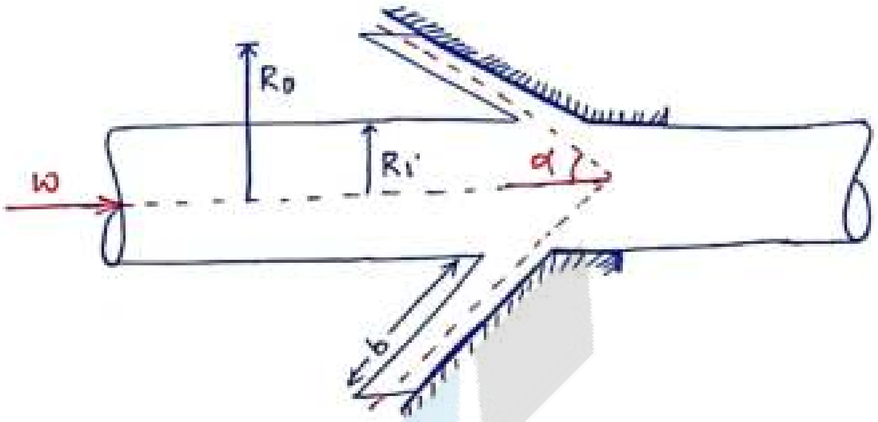
$$\frac{16.3\times10^3}{4\cdot2\pi\cdot(.16)t} \le 25\times10^6$$

$t = 1.59\,mm$

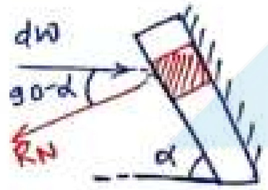
$= 1.6\,mm$

# * CONICAL COLLAR BEARING :-

$\alpha$ = Semi cone angle




$$\boxed{b = \frac{R_o - R_i}{\sin\alpha}}$$




$dF_f = \mu R_N$

$$dF_f = \frac{\mu\, dW}{\sin\alpha}$$

$R_N \cos(90-\alpha) = dW$

$$\boxed{R_N = \frac{dW}{\sin\alpha}}$$

| Flat collar | | Conical collar |
|---|---|---|
| $\mu$ | $\longrightarrow$ | $\dfrac{\mu}{\sin\alpha}$ |

## Multi cone collar Bearing (UPT)

$$P_{ind} = \frac{W}{n\pi(R_o^2 - R_i^2)}$$

$$T_f = \frac{2}{3}\frac{\mu}{\sin\alpha} W\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$$

# JOURNAL BEARING
## OR
# RADIAL LOAD BEARING

$R$ = radius of Shaft / Journal

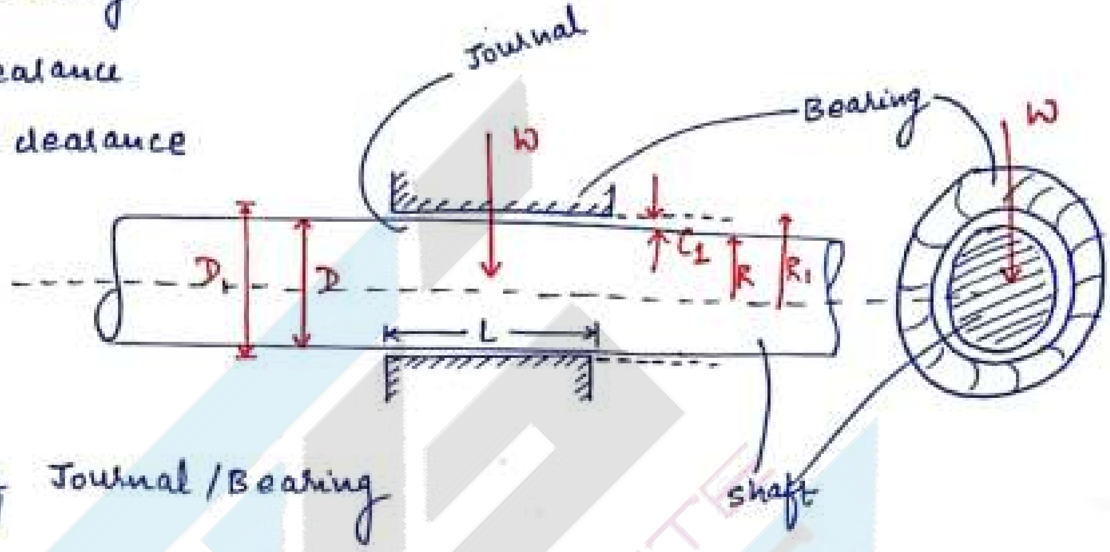
$R_1$ = radius of Bearing

$D$ = Dia of shaft / Journal

$D_1$ = Dia of Bearing

$C_1$ = Radial clearance

$C$ = dimetrical clearance




$C_1 = R_1 - R$

$C = D_1 - D$

$C = 2C_1$

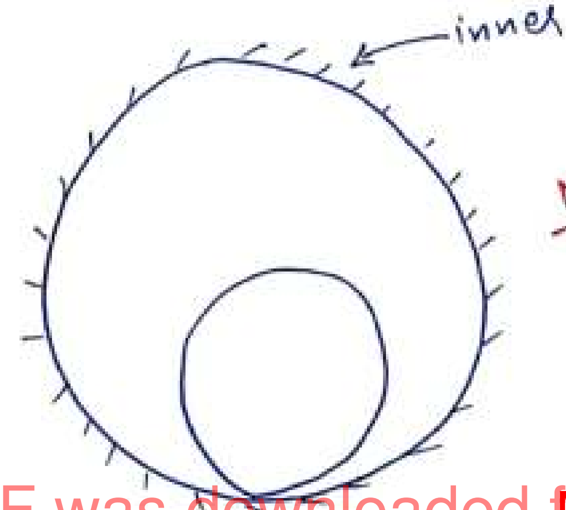
$L$ = length of Journal / Bearing

Journal Bearing is a Radial load sliding contact bearing, generally operating with hydrodynamic lubrication.

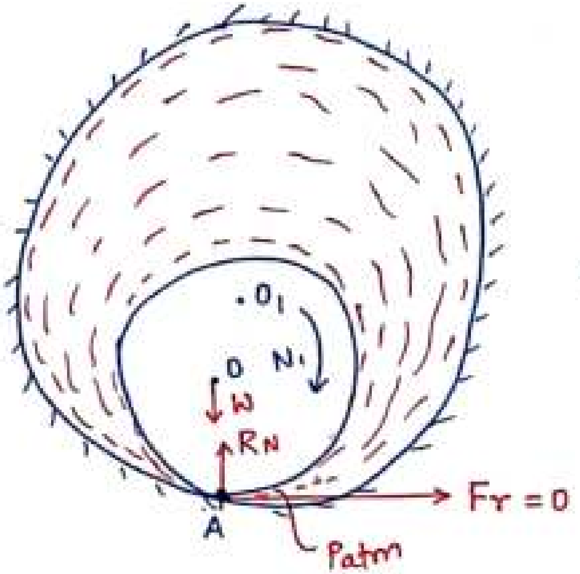
## # HYDRODYNAMIC LUBRICATION :-

Case I  Stationary condition  $N_1 = 0$

$$\boxed{R_N = W}$$




N.P.

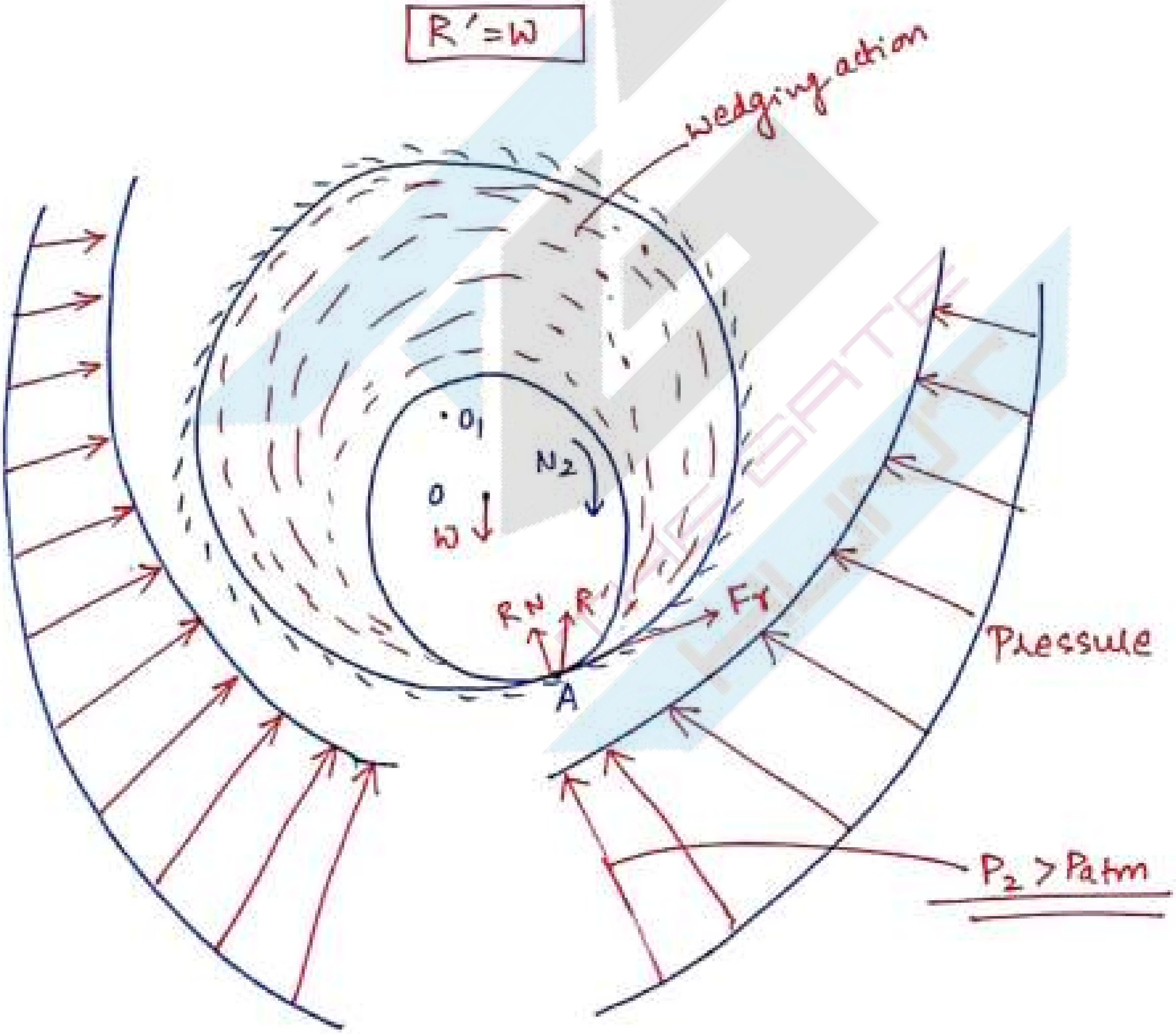



$N_1 = 0$

Case II

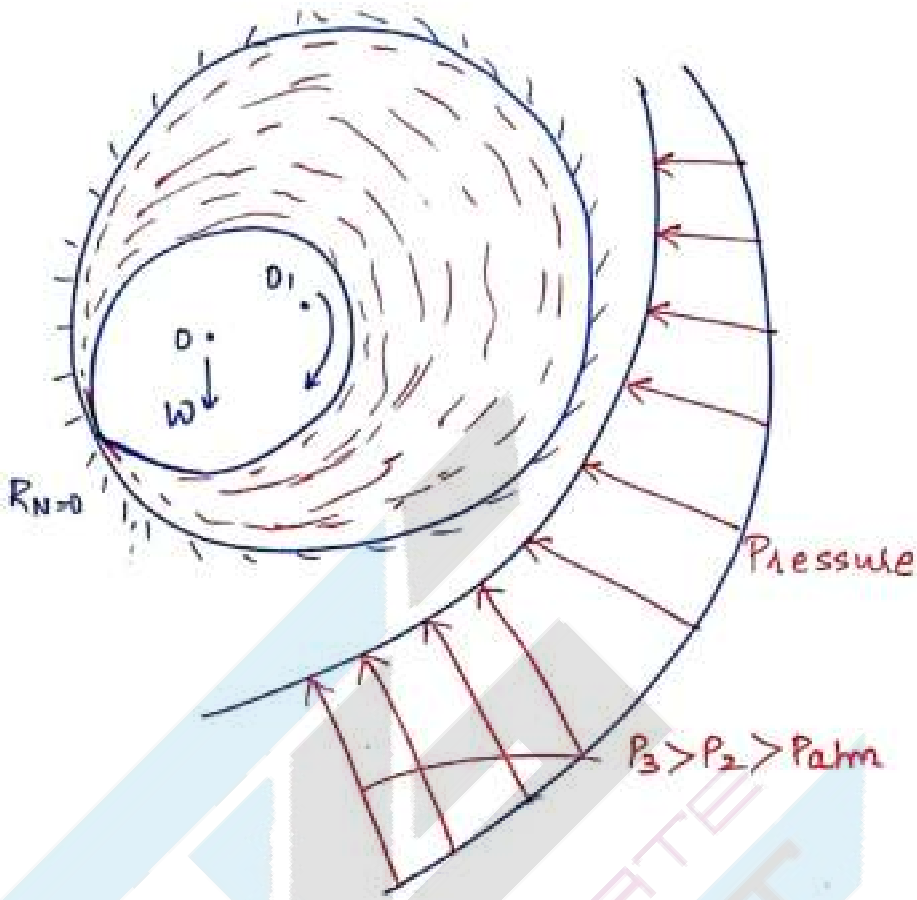
$N_2 > N_1 = 0$

$\boxed{R' = W}$



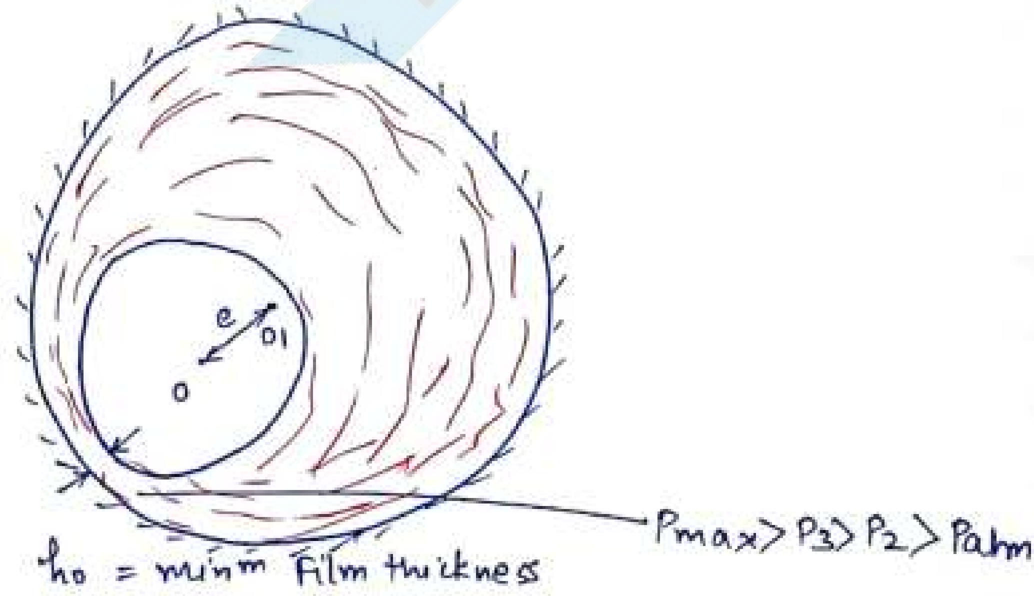
Case III $N_3 > N_2 > N_1 = 0$

[just lift)]




Case IV $N_{max} > N_3 > N_2 > N_1 = 0$

J.B. at Dynamic condn. or J.B. at maxm. Running condn.



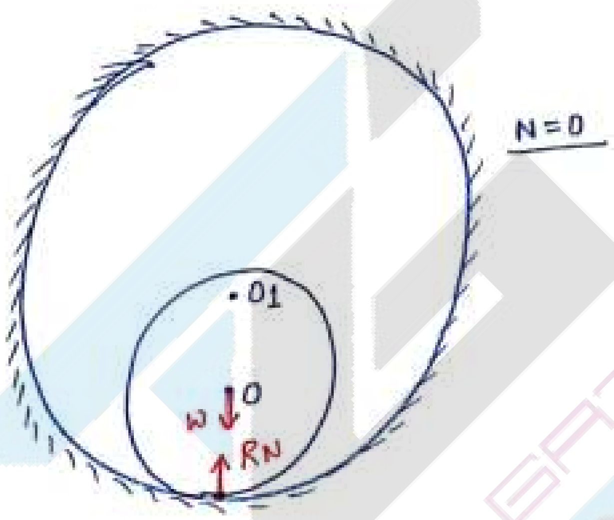
Conclusions → (1) due to rotation of the shaft, this lubricant is converted into a sticky film and as this film enters from wider space to narrow space, pressure of the lubricant becomes maximum. This phenomenon for lubricant is known as wedging action / dynamic action / convergent action of the lubricant.

(2) and this high pressure is responsible for lift the shaft.

# HYDROSTATIC LUBRICATION :→

Case I stationary condn.

[without lubrication]




$W_{pump\,open} = -\int v\,dP$

$m = \rho V$

$\rho = 1/v$ → specific vol^m

$W_{open} = -\int \frac{dP}{\rho}$

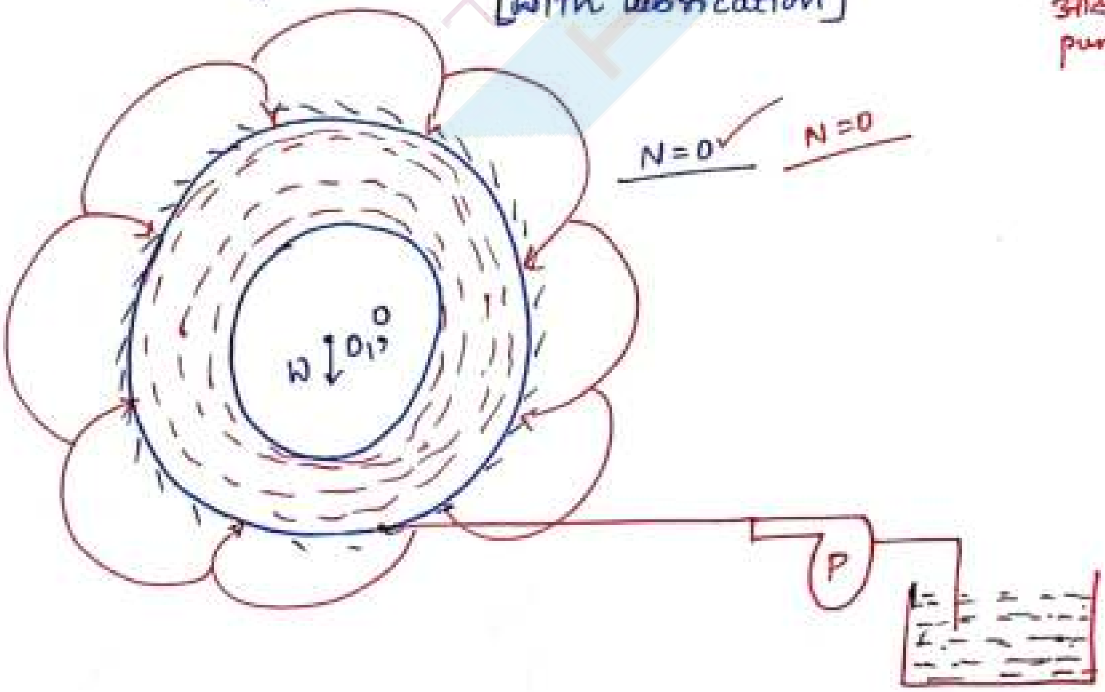
$W_P = -\int \frac{dP}{\rho_l}$ | $W_c = -\int \frac{dP}{\rho_g}$

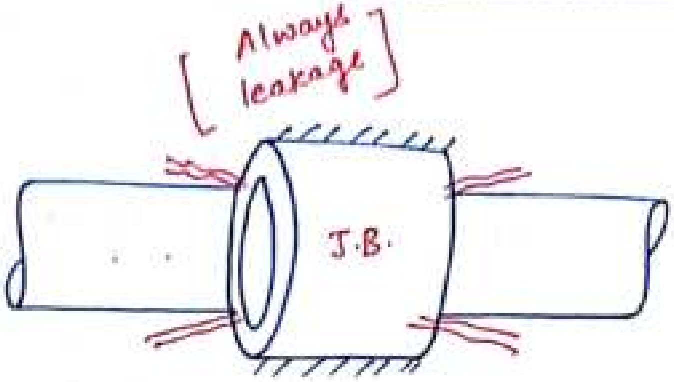
$\rho_l >>>>> \rho_g$

$W_P <<<< W_c$ → दुनिया का सबसे आसान काम pump चलाना

Case II stationary condn.

[with lubrication]






Note :- Required continuous lubrication

Continuous lubrication ⟶ Journal Bearing whether Hydrostatic or hydrodynamic (B).

## * TABLE

| HYDRODYNAMIC Lubrication | HYDROSTATIC Lubrication |
|---|---|
| 1) lubricant is supplied into the Bearing at atmospheric pressure. | ① lubricant is supplied into the Bearing at higher pressure. |
| 2) Pressure of the lubricant ↑ses due to wedging action or convergent action of the lubricant | ② Pressure of the lubricant ↑ses by an external devices like pump. |
| ③ Metal to Metal contact will avoided only at high speed condition. | ③ M-M contact will avoided at stationary condn. only. |
| 4) Motion of the shaft is eccentric wrt the Bearing housing. | ④ Motion of the shaft is concentric wrt the Bearing housing. |
| 5) starting Torque is high | ⑤ starting Torque is less. |
| 6) Cost of the lubrication less | ⑥ Cost of the lubrication more. |

(7) It is used in a application where shaft is subjected to lighter load at stationary condition.

(8) It is used in a application like IC Engine crankshaft, steam and gas turbines, electrical Motors, generators, etc.

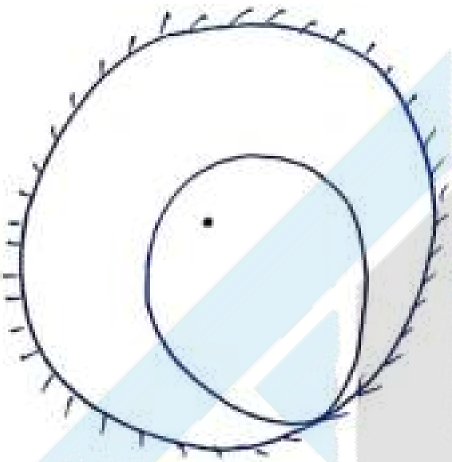
(7) It is used in application where shaft is subjected to heavier load at stationary condition.

(8) It is used in heavy machinery like ~~Ball~~ Bowl powermills in thermal power plant, vertical turbogenerators, large concrete mixtures, etc.

now from here, hydrodynamic discussion mein Rahega........

# FRICTIONAL TORQUE & POWER LOSS in Journal Bearings
(concept of friction circle)

MOHIT KUMAR CHOUKSEY



Redraw

Point of contact shifts towards Right side due to friction $F_r$.

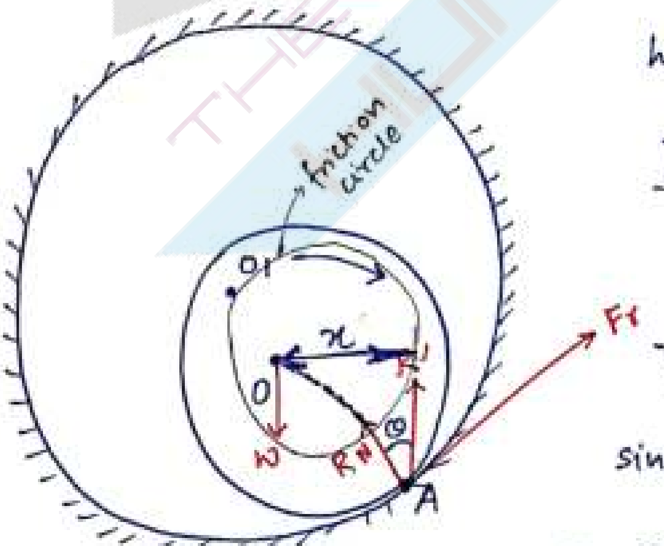
The responsible factor for couple is friction $F_r$.

This couple is Twisting couple.

$\boxed{R' = W}$

$O_1 \to$ centre of Bearing

$O \to$ centre of shaft




hence,

$T_f$ = frictional torque

$T_f = Wx$

$\sin\theta = \frac{x}{R}$

$x = R\sin\theta$

$T_f = WR\sin\theta$

$\theta \to$ very less

$\sin\theta = \theta = \tan\theta$

here, case no. II in discussion →

I posn.

$R'$, W

couple, shaft — opposing

$x$ = friction circle Radius

$\boxed{T_f = \mu WR}$,

$R_{eff} = R$

$\boxed{T_f = WR\tan\theta}$ $\boxed{\tan\theta = \mu}$

$P_{loss} = T_f \times \omega$

$P_{loss} = \mu W \underbrace{R \times \omega}$ $= \mu W V$

$$\boxed{P_{loss} = \mu W V}$$

Conclusion → ① when shaft is in stationary condn. (absence of friction), the normal Reaction offered by the Bearing is inline with line of action of the load acting from the journal

② When shaft is in motion, (presence of friction), the Resultant ($R'$) (Resultant of friction ($F_r$) and normal Rxn. $R_N$) is deviated by a distance '$x$' from the line of action of the load acting from the journal. and this '$x$' is known as Friction circle Radius.

③ A circle drawn from the centre of the shaft by taking Radius '$x$' is known as friction circle.



dependency of $x$

$x = R \sin\phi$

$x = R \cdot \mu$

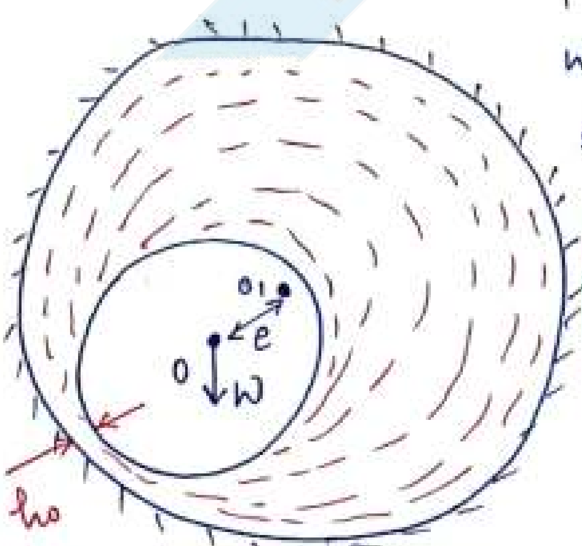
$$\boxed{x = f[R, \mu]}$$

* Terminology used :- for journal Bearing

$$\boxed{\begin{aligned} C_1 &= R_1 - R \\ C &= D_1 - D \end{aligned}}$$

const. रहेगा
speed अगर
change नहीं होगा

ho is not a clearance




ho

minm. Film thickness

speed ↑ → ho ↑ → e ↓ → ε ↓

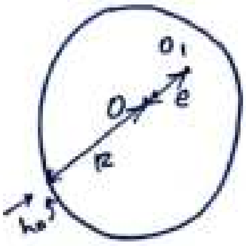
W ↑ → ho ↓ → e ↑ → ε ↑

Z ↑ → ho ↑ → e ↓ → ε ↓

Temp ↑ → Z ↓ → ho ↓ → e ↑ → ε ↑

→ viscosity of fluid ↑, Pressure genrr capacity of fluid ↑.

① **Eccentricity** → is the distance b/w centre of the shaft and centre of the Bearing.




$$\boxed{e + R + h_o = R_1}$$

$$e = \underbrace{R_1 - R}_{} - h_o$$

$$e = C_1 - h_o$$

$$\boxed{e = \frac{C}{2} - h_o}$$

clearance (if D is given)

② **Eccentricity Ratio** → It is defined as the Ratio of eccentricity to **($\epsilon$)** Radial clearance.

$$\epsilon = \frac{e}{C_1}$$

attitude

$$\epsilon = \frac{C/2 - h_o}{C/2}$$

$$\boxed{\epsilon = 1 - \frac{2h_o}{C}}$$



③ **Bearing clearance** → $C_1$ = Radial Clearance.

$C$ = diametral clearance / clearance

(if only it is given then assume ($C$))

Diameter to clearance Ratio ⇒ $D/C$.

Radius ——— ,, ——— ⇒ $R/C_1$.

④ **$C/D$ Ratio** →
or
**$D/C$ Ratio** → if $C\downarrow \rightarrow C/D\downarrow \rightarrow P_{loss}\uparrow \rightarrow W\uparrow$
desi' undesirable

$C\uparrow \rightarrow C/D\uparrow \rightarrow P_{loss}\downarrow \rightarrow W\downarrow$

$$\boxed{\begin{array}{c} .001 \le C/D \le .002 \\ \text{(or)} \\ 500 \le D/C \le 1000 \end{array}}$$

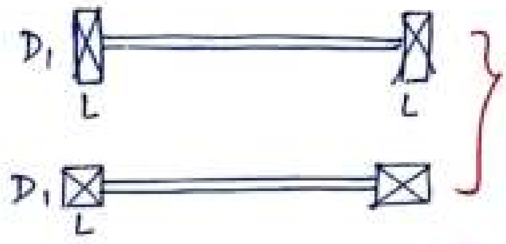
⑤ $\frac{L}{D_1}$ Ratio →

$D_1$ → Bearing diameter

से define ki Bearing keliye hua hai.

Short Bearing ← $\frac{L}{D_1} < 1$ ⇐

Square Bearing ← $\frac{L}{D_1} = 1$ ⇐




long Bearing ← $\frac{L}{D_1} > 1$ ⇐


| | Point | UDL |
|---|---|---|
| simply supported $\delta \Rightarrow$ | $\frac{WL^3}{48EI}$ | $\frac{5WL^4}{384EI}$ |
| Fixed support $\Rightarrow \delta \Rightarrow$ | $\frac{WL^3}{192EI}$ | $\frac{WL^4}{384EI}$ |

Beam में support hote hain, shaft में support नहीं होते only Bearing is acting as a support for shaft, so please refer $L/D_1$ Ratio of Bearing.

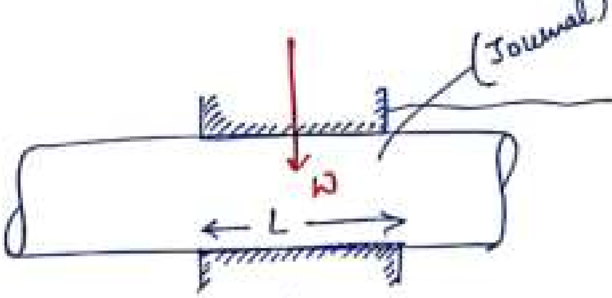
⑥ Bearing Pressure :- (compression)

⇓

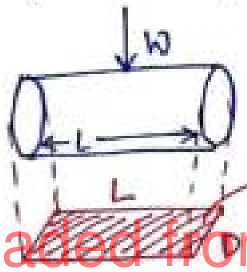
comp^n

(compr. / Bearing / crushing)

(Tension / Tearing)

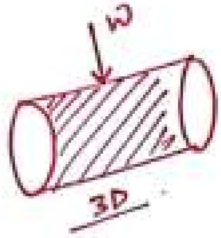



→ earth / found^n never fails due to load.

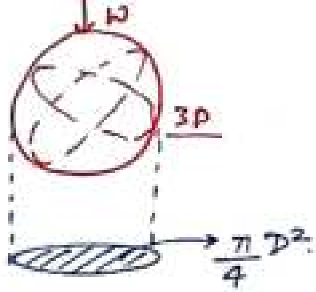



Projected area ⇒ LD

* $\boxed{P_{ind} = \frac{W}{LD}}$




$$\boxed{P_{ind} = \frac{\text{load}}{\text{projected area}}}$$




$$\boxed{P_{ind} = \frac{W}{LD}} \;**$$

safe condition

$$P_{ind} \le P_{per}$$

$$\frac{W}{LD} \le P_{per} \leftarrow \text{property of material}$$

$$\boxed{W_{max} = LD \cdot P_{per}}$$

$\Downarrow$

Strength of Journal Bearing.

⑦ Power loss

$$P_{loss} = \mu W V$$

$$\boxed{\mu = ?}$$

⑧ Coefficient of friction →

Mcc Kee's Eqn.

$$\mu = f\left[\left(\frac{Zn}{P}\right), \left(\frac{D}{C}\right), \frac{L}{D_1}\right]$$

$\frac{Zn}{P} \rightarrow$ Bearing characteristic Number.

$Z \rightarrow$ viscosity (absolute) of the lubricant. (dynamic viscosity) $\left(\frac{N\text{-}s}{m^2}\right)$

$$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right) + K \quad \left[\text{Petroff equation}\right]$$

$K$ is a fn. of $L/D_1$

$$\boxed{\begin{array}{l} \text{imp } ** \; n \rightarrow \text{speed "Revolution per sec"} \\ \cancel{\text{rad/s}} \quad \cancel{\text{N}} \end{array}}$$

$$\omega = 2\pi n$$

P = Bearing pressure

$$P = \frac{W}{LD}$$

K → Leakage Factor

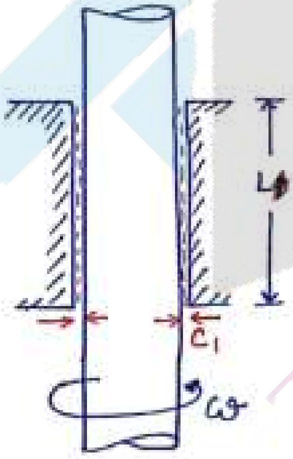
K depends on $L/D$

$$\left[\begin{array}{l} \underline{\underline{K = .002}} \Leftarrow L/D_1 < .75 \\ \underline{\underline{K = .003}} \Leftarrow 2.8 \geq L/D_1 \geq .75 \end{array}\right]$$

$$\boxed{\mu = 2\pi^2 \left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right)}$$

** when K is given then use (by adding) $\underline{K}$. otherwise not use.

* PROOF :-




$$\tau = Z \frac{dV}{dh} \quad \text{(by F.M.)}$$

$$\tau = \frac{Z \cdot V}{C_1}$$

$$\frac{F_s}{A_s} = \frac{Z \cdot \pi D n}{c/2}$$

$$\frac{F_f}{\pi D L} = \frac{Z \pi D n}{\frac{c}{2}}$$

$$F_f = \frac{\pi D L Z \pi D n}{c/2}$$

$$T_f = F_f \times R$$

$$T_f = \frac{\pi D L Z \pi D n \times R}{c/2} \quad \text{—(1)}$$

$\boxed{T_f = \mu W R}$ by friction circle ①

$$\mu W \cancel{R} = \frac{\pi D L Z \cdot \frac{\pi D n}{60} \cancel{R}}{\frac{c}{2}}$$

$$\boxed{\mu = 2\pi^2 \left(\frac{Zn}{p}\right)\left(\frac{D}{c}\right)}$$

# Sommerfield No. :- Sommerfield No. Remains constant for a given Bearing hence it is used to corelate the working conditions of different m/c's which are operating with same Bearing.

$$\boxed{S = \frac{Zn}{p}\left(\frac{D}{c}\right)^2}$$

Pa.s, 1/s, Pa

(unitless)

$\boxed{S_1 = S_2}$ for a given Bearing

Gate-Box

$$T_f = \mu W R$$

$$P_{loss} = \mu W V$$

$$P_{ind} = \frac{W}{LD}$$

$$\mu = 2\pi^2\left(\frac{Zn}{p}\right)\left(\frac{D}{c}\right)$$

$$S = \frac{Zn}{p}\left(\frac{D}{c}\right)^2$$

Q1 A Natural feed journal bearing of diameter 50mm and length of 50mm operating at a speed 20 rps carries a load of 2 KN. The lubricant used has a viscosity of 20 millipascal-sec and the Radial clearance is 50 micro m, the sommerfield no. of the Bearing is.

Sol

$$S = \frac{Zn}{P}\left(\frac{D}{c}\right)^2$$

$$S = \frac{20\times10^{-3} \times 20}{\cancel{2\times10^3}\ \frac{2\times10^3}{(.05)^2}}\left(\frac{0.050}{\cancel{50\times10^{-6}}\ 100}\right)^2 = 1.25$$

W

Doubt → Bearing D = 50mm

D = $D_1 - c$ = 49.9 mm

$\frac{D}{c} \approx 1000$ obj → $\boxed{D_1 \approx D}$

Q A Journal Bearing has a shaft dia. of 40mm and length of 40mm. The shaft is rotating at 20 Rad/sec and the viscosity of the lubricant is 20 milli pa-sec, the Radial clearance is .020 mm, the loss of Torque due to viscosity is :-

Sol

D = 0.040 m  r = 20 Rad/sec  $\mu = 20\times10^{-3}$ Pa

L = 0.040 m  c = 0.020 mm

$$S = \frac{Zn}{P}\left(D/c\right)^2 =$$

$\times \frac{20}{20}$

$4\times10^{-6}$

$$\therefore T_f = \mu W R = 20\times10^{-3} \times W \times .020$$

$$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(D/c\right)$$

$$20\times10^{-3}\times W \times .020 = 2\pi^2 \frac{(20\times10^{-3} \times 20\ \text{rad/sec})}{P}$$

Hint→ $T_f = \mu W R$

$P_{ind} = \frac{W}{LD}$

$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(D/c\right)$

$$T_f = 2\pi^2 \frac{Zn}{\frac{W}{LD}}\left(\frac{D}{c}\right)W \cdot R$$

$$T_f = 2\pi^2 (Zn\, LD)\left(D/c\right) R$$

$$T_f = 2\pi^2\left(20\times10^{-3}\times\frac{20}{2\pi}\right)(.04)^2\left(\frac{.04}{.04\times10^{-3}}\right)(.02) = \underline{.04\,N\text{-}m}$$

Q A lightly loaded Full journal Bearing has Journal diameter of 50mm and Bush Bore of 50.05mm. and the Bush length is 20mm. If the Rotational speed of the Journal is 1200 rpm, the power loss in watt if the avg. viscosity of the lubricant is 0.3 Pa-s.

Sol $D = 50mm$    $n = 1200 rpm$

$d = 50.05mm = 0.050m$    $\mu = 0.3\,Pa\text{-}s$

$L = 20mm = 0.020m$

$T_f = 0.3\times W\times 0.025m$

$T_f = 75\times10^{-4} m\,W$

$P_{ind} = \frac{W}{0.020\times0.0505}$

$= \frac{W}{1010}$

$P_{ind} = \frac{W}{10^{-7}\times1010}$

$\frac{505 \times 2}{1010}$

Sir

$$P_{loss} = 2\pi^2\frac{Zn}{\frac{W}{LD}}\left(\frac{D}{c}\right)WV$$

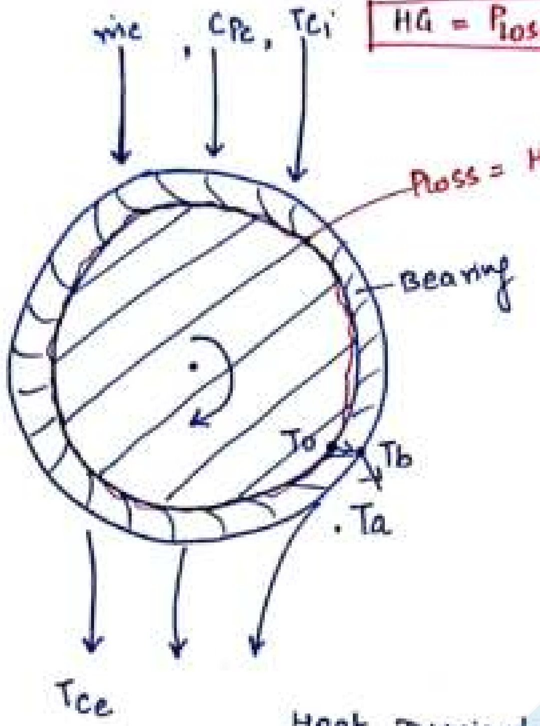
$$P_{loss} = 2\pi^2 Zn(LD)\left(\frac{D}{c}\right)\frac{\pi DN}{60}$$

$$= 2\pi^2 .03\times\frac{1200}{60}(.02\times.05)\left(\frac{.05}{.05\times10^{-3}}\right)$$

$$\times\left(\frac{.05\times1200}{60}\right) \Rightarrow \underline{37.2kW}$$

# Mass flow Rate of the coolant Required in Journal Bearing →

$$\boxed{H_G = P_{loss} = \mu W V}$$




$T_o$ = maxm. tempr. that lubricant can bear without any failure

$T_b$ = outer surface tempr. of the Bearing

$T_a$ = ambient tempr / Room tempr.

Heat Dissipation at the worst condn.

$$H_D = h A \Delta T$$

$$H_D = h \pi D L (T_b - T_a)$$

$\pi h = C_D$ Heat dissipation coefficient

$$\Downarrow$$

$$\frac{W}{m^2 K}$$

$$H_D = C_D . D L (T_b - T_a)$$

$$\boxed{(T_b - T_a) = \frac{T_o - T_a}{2}}$$

Let us assume

***

$$\boxed{H_D = C_D . D L \frac{(T_o - T_a)}{2}}$$

if $\left.\begin{array}{l} H_D > H_G \\ H_D = H_G \end{array}\right]$ No - coolant

$H_G > H_D$ ] used coolant

$HG - HD = \dot{m}_c \, Cp_c \, (\Delta T)_{coolant}$

$\boxed{\dot{m}_i = \text{known}}$

<u>Note</u> :- Monitoring of the journal Bearing is done by measuring inside tempr by thermometer and measuring inside vibrn. by accelerometers.

Q VIMP 2017 A natural feed journal Bearing has a shaft dia. of 50mm and length of 50mm operates at a speed of 900 rpm. The Bearing is lubricated with an oil whose absolute viscosity & operating tempr. are 0.03 Pa-s and 75°C. Assume $(D/c)$ Ratio → 1000, $C_D = 600 \frac{W}{m^2\text{-}K}$. Room tempr. 25°C. Find out :- ⓐ Rate of artificial cooling Required. ② find out the mass flow rate of the coolant req. if $(Cp)_{coolant}$ is 1.8 KJ/kg-K and tempr. diffr. for coolant for outlet and inlet is 15°C.

<u>Sol</u> $D = 0.050$ m $\qquad N = 900$ rpm $\qquad T_B = 75°C \qquad C_D \rightarrow 600 \frac{W}{m^2K}$

$L = 0.050 \qquad Z = 0.03$ Pa-s $\qquad D/c \rightarrow 1000 \qquad T_a = 25°C$.

$HG - HD = \dot{m}_c \, Cp_c (\Delta T)_{coolant}$

<u>HD = 37.5W.</u> $\qquad HG = ? \qquad HG = \mu W V$

$= \frac{\pi D N}{60} \times W \times \mu$

---

$HG = P_{loss} = 2\pi^2 Z n (LD)\left(\frac{D}{C}\right) \frac{\pi D N}{60}$

$HG = 2\pi^2\left(.03 \times \frac{900}{60}\right)(.05)^2 (1000) \frac{\pi(.05)(900)}{60}$

$HG = 52.32W$

$HD = \frac{C_D DL(T_0 - T_A)}{2} = \frac{600 \times (.05)^2 [75 - 25]}{2}$

$HD = 37.5W$

$HG - HD =$ Artific cooling

$= 14.38W$

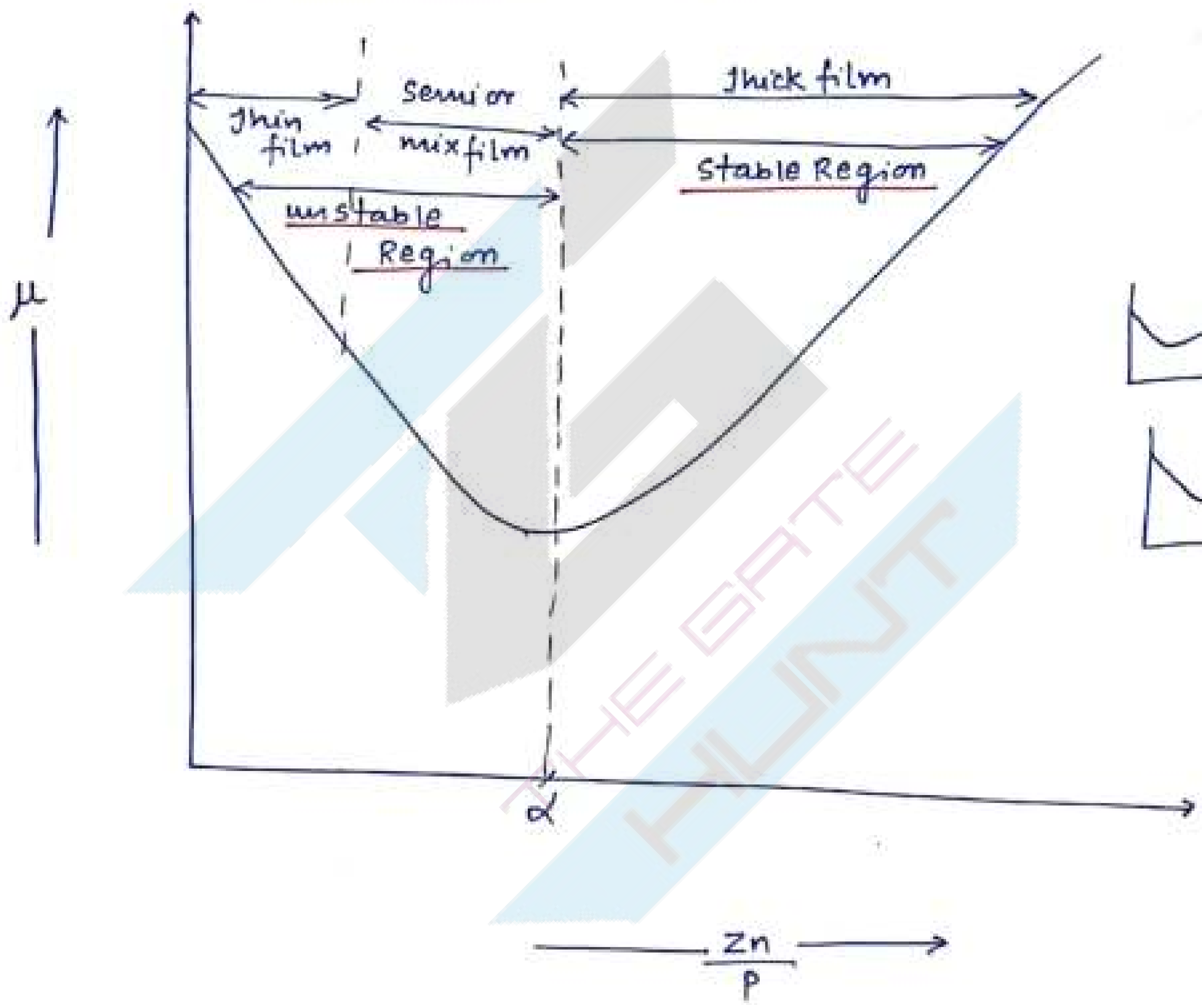
$$14.38 = \dot{m}_c \, C_{p_c} \, (\Delta T)_{coolant}$$

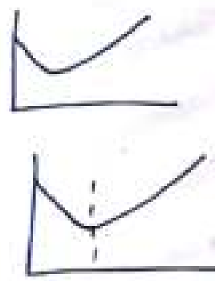
$$14.38 = \dot{m}_c \times 1.8 \times (15)$$

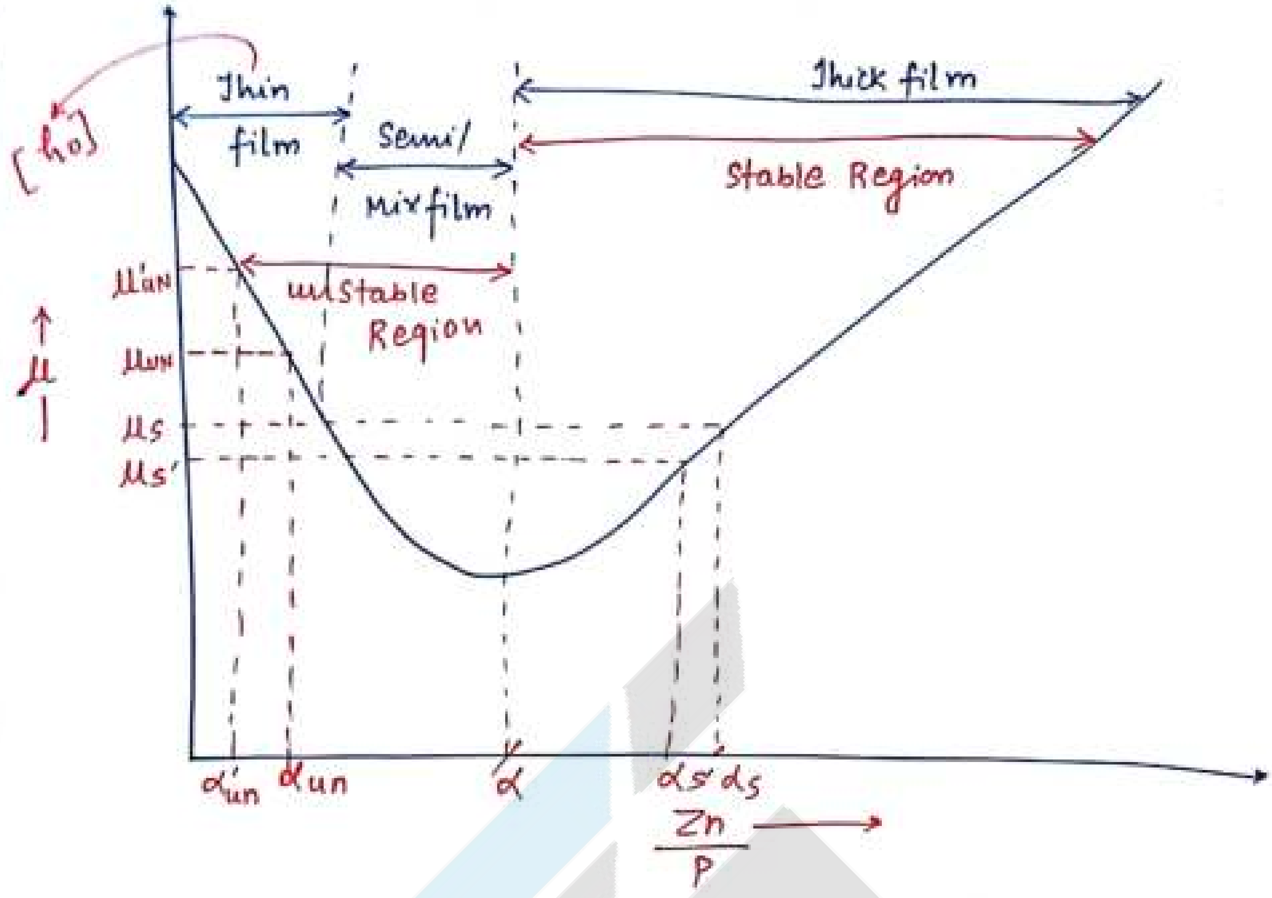
$$\boxed{\dot{m}_c = .54 \text{ gm/sec}}$$

24/11/2016

* Significance of Bearing characteristic No. and Bearing Modulus :-
(McKee's Investigation / Reynolds analogy) :-






**unstable Region**

load↑ → Ploss↑ → HG↑ → Temp↑ → $Z\downarrow$

$\frac{Zn}{P}\downarrow \rightarrow \alpha_{un}\downarrow \rightarrow \mu\uparrow \rightarrow$ Ploss↑ → Temp↑

$\uparrow\uparrow P_{loss} = \mu\uparrow W\uparrow V$


**Stable Region**

load↑ → Ploss↑ → HG↑ → Temp↑ → $Z\downarrow \rightarrow \frac{Zn}{P}\downarrow$

$\alpha_s\downarrow \rightarrow \mu\downarrow \rightarrow$ Ploss↓ → Temp↓

Stable Region

$\downarrow\uparrow P_{loss} = \mu\downarrow W\uparrow V$

$$\underset{\text{stable}}{\mu\downarrow} = 2\pi^2\left(\frac{Zn}{P\uparrow}\right)\left(\frac{D}{c}\right)$$

$\alpha \rightarrow \left(\frac{Zn}{P}\right)_{min}$ for stability

⇓

**Bearing Modulus**

$\left(\frac{Zn}{P}\right)_{working} \rightarrow$ Bearing characteristic No.

Note: hence Bearing charac. No. is always greater than Bearing Modulus.

$$\boxed{3\alpha \le \left(\frac{Zn}{P}\right)_{\substack{\text{working}\\ \text{or}\\ \text{Actual}\\ \text{Design}}} \le 15\alpha}$$

For static load

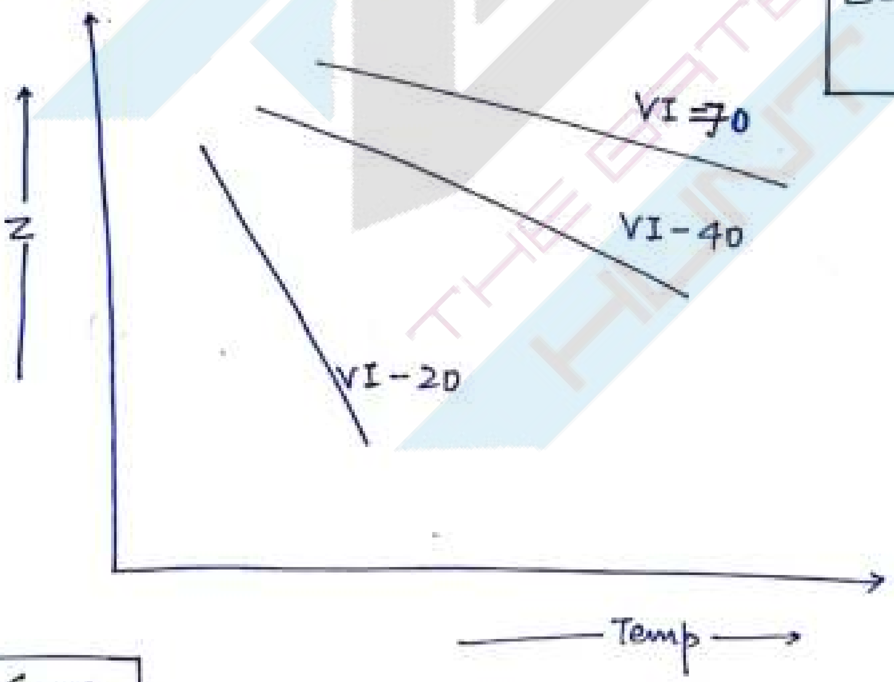
$$\boxed{3\alpha \le \left(\frac{Zn}{P}\right) \le 5\alpha}$$

For high impact /fatigue load

$$\boxed{13\alpha \le \left(\frac{Zn}{P}\right) \le 15\alpha}$$

* VISCOSCITY INDEX → It represent the Rate of change of viscosity with respect to the temperature.

$$\boxed{Z = A + \frac{B}{Temp}}$$

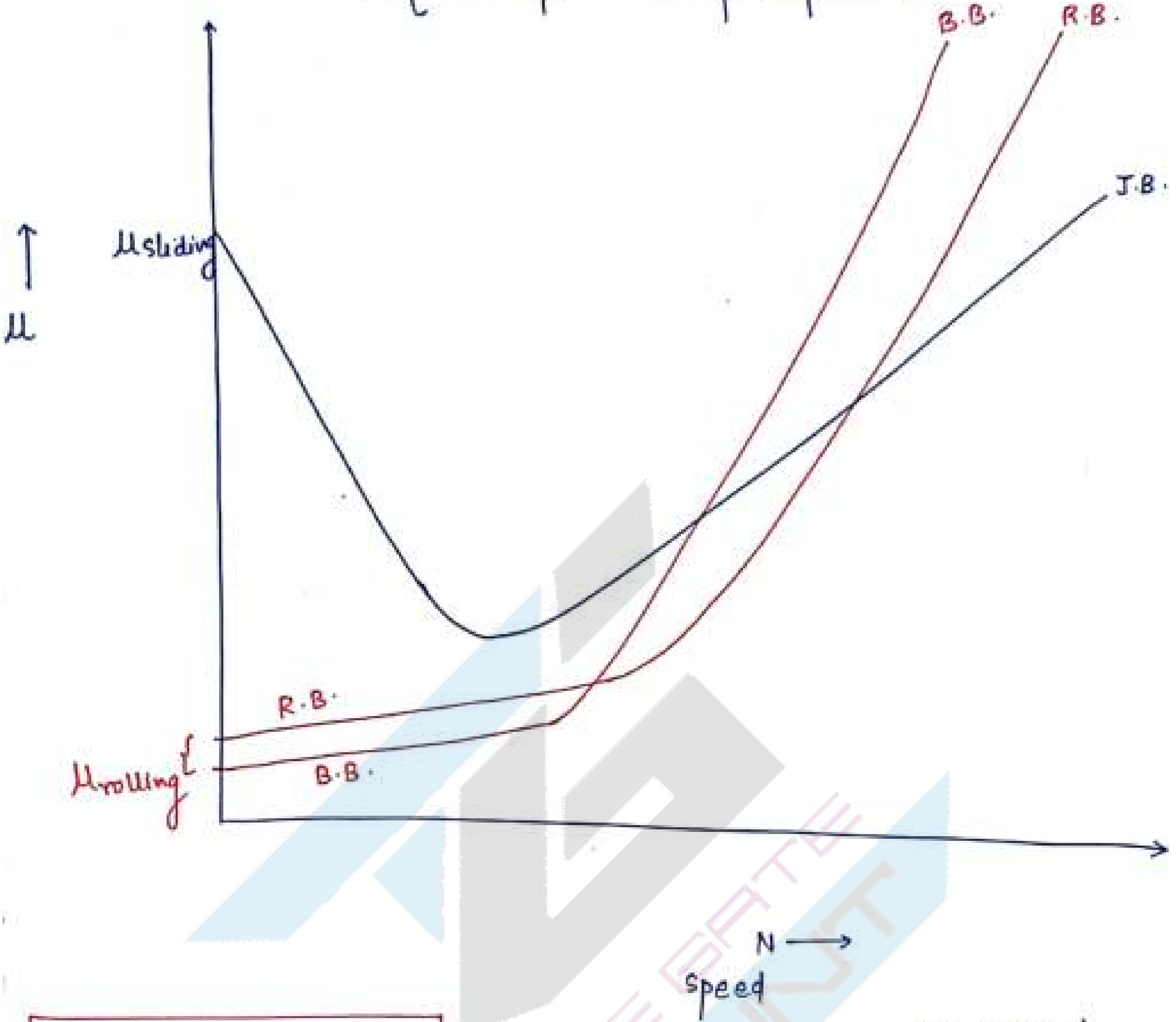



$$\boxed{0 \le VI \le 100}$$

VI is not related to ~~sss~~ stability

Note (∵ we can't define the gradual or fast Rate). of ↑ $\mu$.

# ANTI FRICTION BEARING




$\boxed{\mu_{rolling} <<< \mu_{sliding}}$

low speed

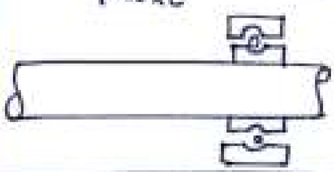
High speed

$\boxed{\mu_{BB} > \mu_{RB} > \mu_{J.B.}}$

sliding highest

| Parameter | J.B. | A.F.B. |
|---|---|---|
| N. | P. | N.P. |

MOHIT KUMAR CHOUKSEY

| Parameter | Journal Bearing JB | made of Babbite material — Anti Friction Bearing AFB (Powder Metallurgy से) |
|---|---|---|
| ① Speed | used for high speed application. | used for low & medium speed application. |
| ② load | only Radial load | radial & axial both |
| ③ Machine service | Machine in continuous service. | Intermittent service required (frequently stopping & starting) |
| ④ Noise | Minm. Noise in all Bearing. | Maxm. Noise in all Bearing. |
| ⑤ Life | life more | life less |
| ⑥ Starting Torque | More | less |
| ⑦ Cost | less | More |
| ⑧ Radial space | less | More  |
| ⑨ Axial space | more | less |
| ⑩ Damping capacity | more | less |

MOHIT KUMAR CHOUKSEY

| | JB | AFB |
|---|---|---|
| ⑪ Lubrication | liquid lubricant & continuous lubrication. | Semi-solid lubricant (grease), → कभी कभार periodic lubrication. |
| ⑫ Type of Failure | sudden failure without any indication. | give indication before failure by making more noise. |
| ⑬ Application | IC Engine crankshaft large motors, concrete mixers, generators etc. steam & gas Turbines. | ~~Steam and gas turbines~~ m/c Tool spindle in lathe Automobile front & Rear axles, gear boxes, small motors, etc. |

# Designation of AFB (Anti friction Bearing)

Type of Bearing ← SKF 6308 → Type of series

08 × 5 = 40mm shaft dia

6 - DGB.B → Deep groove Ball Bearing

3 - Taper Roller Bearing

# Type of Series

" N.P. "

## Type of Series

1 → Extra light series → 100 series

2 → Light series → 200 series

3 → medium series → 300 series

4 → heavy series → 400 series

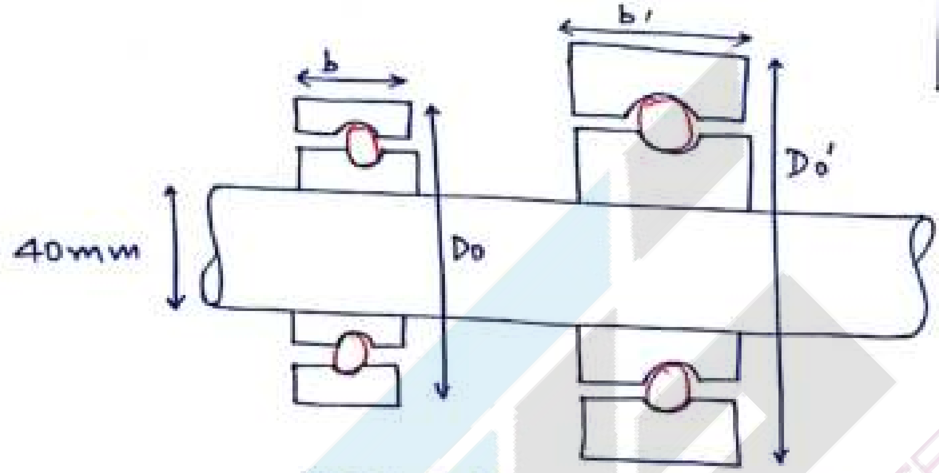
5 → extra heavy series → 500 series

load capacity ↑ →

Dimn. ↑ →

cost ↑ →

$D_o' > D_o$
$b' > b$




SKF 6 3 08 SKF 6 4 08

| * Bearing | Shaft Diameter |
|---|---|
| SKF 6300 → | 10mm |
| SKF 6301 → | 12mm |
| SKF 6302 → | 15mm |
| SKF 6303 → | 17mm |
| | by Rule ×5 |
| SKF 6304 → | 20mm |
| SKF 6305 → | 25mm |

* Indian Standards :-

IS - 40 - BC - 03

shaft dia. → 40

Type of series → 03

Type of Bearing → BC

BC → Deep Groove B.B

* Different terms used while selecting a Series for Antifriction Bearing :-

① Equivalent load → $P_e$ or $P_m = S\left[X V F_r + Y F_a\right]$

S = service Factor / shock factor.

V = Race Rotation factor

X = Radial load factor

Y = axial load factor

$F_r$ = Radial load

$F_a$ = axial load

S :-

steady load / No-shock ⇒ S = 1

light shock ⇒ S = 1.5

Moderate shock ⇒ S = 2

Heavy —"— ⇒ S = 3

Extra heavy —"— ⇒ S = 3.5

V :-

Inner race rotate ⇒ V = 1

outer race rotate ⇒ V = 1.2

X, Y :

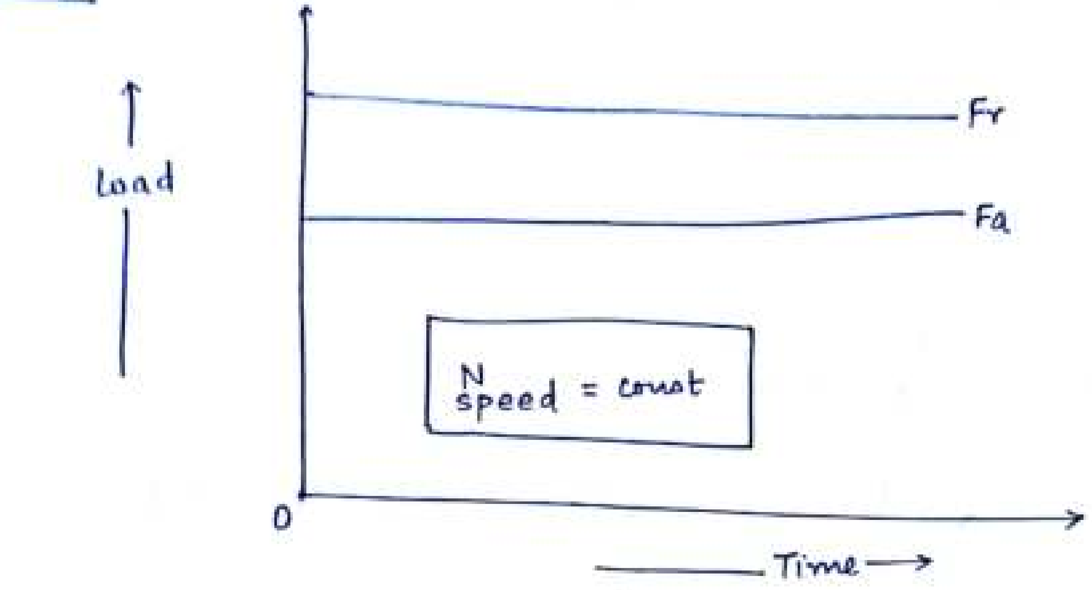
Thrust B.B → X = 0, Y = 1

cyl. R.B. → X = 1, Y = 0

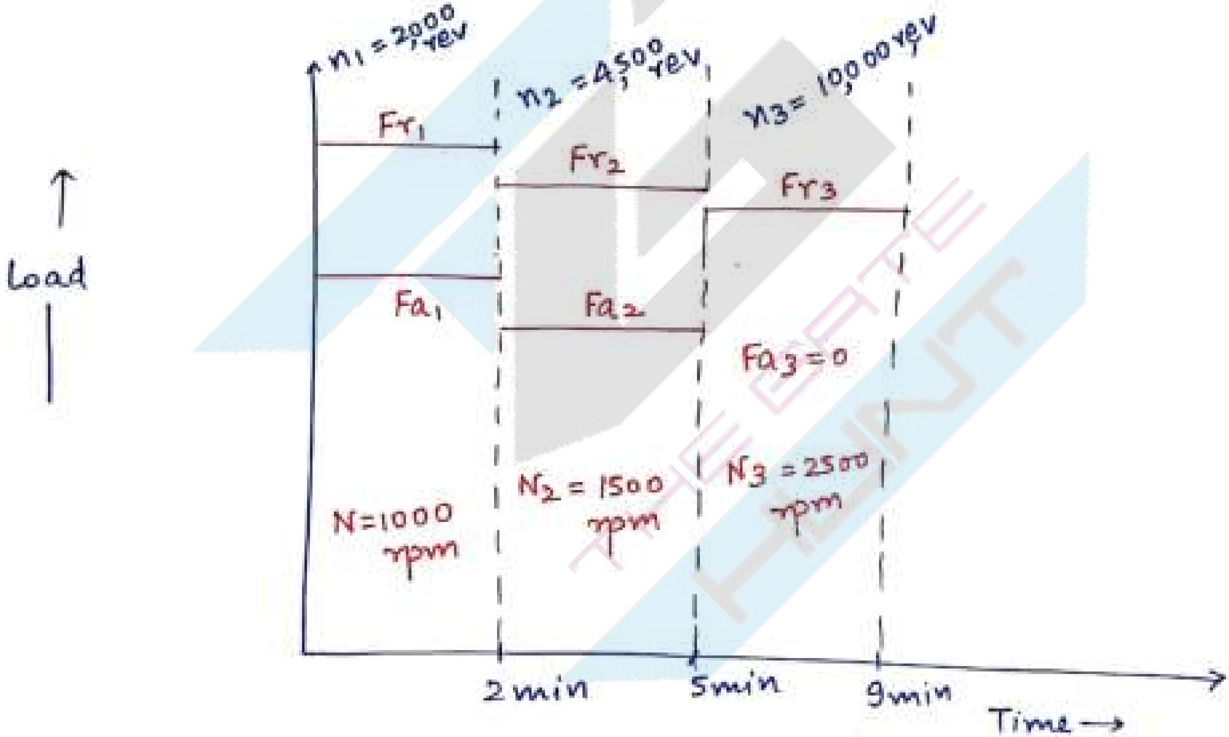
DGBB, TaperB.B → X > Y

Angular contact B.B. → Y > X

**NOTE :-**




The above formulae for equivalent load is only valid when load and speed remains constant with respect to the time.




$$Pe_1 = S_1 [X_1 V_1 Fr_1 + Y_1 Fa_1]$$

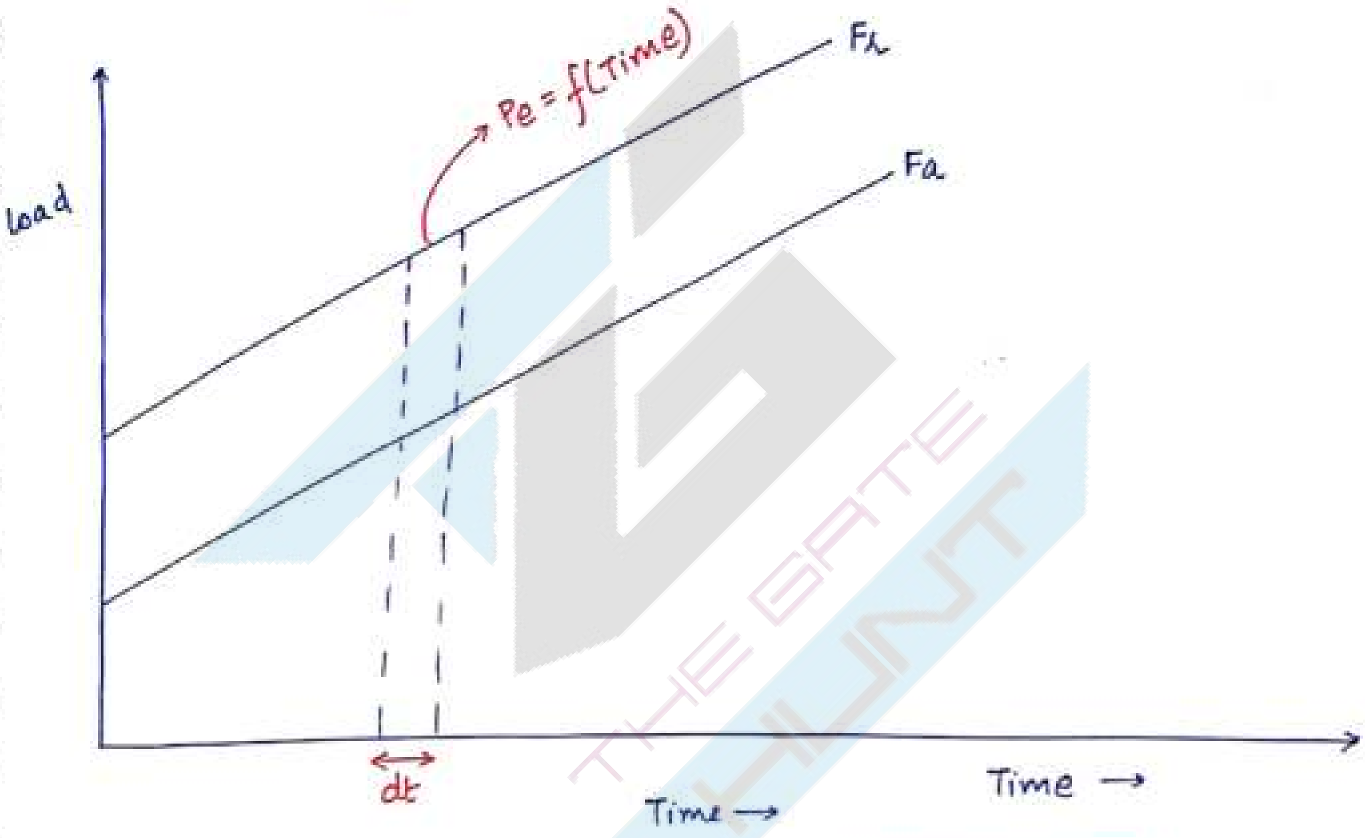
$$Pe_2 = S_2 [X_2 V_2 Fr_2 + Y_2 Fa_2]$$

$$Pe_3 = S_3 [X_3 V_3 Fr_3]$$

$$P_e = \sqrt[3]{\frac{n_1 P_{e_1}^3 + n_2 P_{e_2}^3 + n_3 P_{e_3}^2 + \cdots}{n_1 + n_2 + n_3 + \cdots}}$$

$n_1, n_2, n_3$ are the no. of revolution that a Bearing has undergone in respective region time $t_1, t_2, t_3, \ldots$

$$P_e = \sqrt[3]{\frac{2000\, P_{e_1}^3 + 4500\, P_{e_2}^3 + 10000\, P_{e_3}^2}{16500}}$$




$$\boxed{P_m \text{ or } P_e = \left[\frac{\int P_e^3\, dn}{\int dn}\right]^{1/3}}$$

* **Life of Antifriction Bearing** → is defined as No. of Revolutions that a Bearing has undergone Before the evidence of first fatigue failure either in the Races or in the rolling element.

(Gate → most imp. part)

Fatigue fail ⇒ No yielding
↳ fracture (crack)

$\boxed{\text{Life} = \text{no. of Revolutions}}$ $= 2000 \times 60 \times 600$

$= 72 \times 10^6$ Revo

2000hr, 600Rpm

* **Nominal Life / Rated Life / Life / $L_{90}$ / $L_{10}$ / Life with 90% Relaibility** :-

→ Always defined for group of identical Bearing

→ Nominal life or Rated life of group of identical Bearing is defined as the no. of Revolution that 90% of this group of — " — " — can serve or exceed at a given speed without any failure.

✓ **Relationship b/w L and $L_{90}$** :-

$$\frac{L}{L_{90}} = \left[\frac{\ln(1/R)}{\ln(1/R_{90})}\right]^{1/1.17}$$

$\boxed{L_{60} = 3.85\, L_{90}}$

$$\frac{L_{60}}{L_{90}} = \left[\frac{\ln(1/.6)}{\ln(1/.9)}\right]^{1/1.17}$$

Ex :- $\frac{L_{50}}{L_{90}} = \left[\frac{\ln(1/.6)}{\ln(1/.9)}\right]^{1/1.17}$

$$\boxed{L_{50} = 5 L_{90}}$$

Half life or Avg. life

Remember → hence, the half life is 5 times the Nominal life.

$\boxed{1000 \text{ Bearing}}$

$L_{10} = L_{90}$

$L_{20} = L_{80}$

$L_{30} = L_{70}$

$L_{40} = L_{60}$

$L_{50} = L_{50}$

Ex :- $\frac{L_{20}}{L_{90}} = \cancel{\left[\frac{\ln(1/.2)}{\ln(1/.9)}\right]^{1/1.17}}$

valid only when > 50%

$$\frac{L_{20}}{L_{90}} = \frac{L_{80}}{L_{90}} = \left[\frac{\ln(1/.8)}{\ln(1/.9)}\right]^{1/1.17}$$

also $\cancel{\frac{L_{20}}{L_{90}} = \left[\frac{\ln(1/.2)}{\ln(1/.9)}\right]^{1/1.17}}$

* **Dynamic load Capacity or Dynamic load Rating (C)** :- It is defined as the max$^{m}$ value of the load that 90% group of identical Bearing can serve for a min$^{m}$ life of 1 million Revolution.

A Dynamic load capacity remains constant for a given Bearing.

$P_e = C \Rightarrow L_{90} = 1MR$

Real load

Real life

Life = $72 \times 10^6$ rev

Ex

$P_e > C \Rightarrow L_{90} < 1MR$

$P_e < C \Rightarrow L_{90} > 1MR$

"72"

$$\boxed{L_{90} = \left[\frac{C}{P_e}\right]^K}$$

[M.R.]

million Revolution

$K = 3 \rightarrow$ Ball Bearing.

$K = \frac{10}{3} \rightarrow$ Rollar Bearing.

$$\left\{ \left[\frac{C}{P_e}\right] = \text{Loading Ratio} \right\}$$

GATE-BOX

$$\boxed{L_{90} = \left(\frac{C}{P_e}\right)^K}$$

$K = 3 \longrightarrow$ B.B.

$K = 10/3 \longrightarrow$ R.B.

Ch $\rightarrow$ 4 Pg 215

4.8 6306

$6 \times 5 = 30$ mm dia

$W = 22$ KN

$N = 600$ rev/min

$T = 2000$ hrs

$P_e = F_r \uparrow + F_a \downarrow$

max 0

$$\boxed{F_{r\,max} = P_e}$$

$L_{90} = 2000 \times 60 \times 600 = 72$ MR

$C = 22$ kN

$L_{90} = \left(\frac{C}{Pe}\right)^3$

$72 = \left(\frac{22}{Pe}\right)^3$

$Pe = 5.28$ KN

4.12 $F = 8000$ hr

$8000 \times 60 \times 60$

$\begin{array}{r} 48 \\ \times 6 \\ \hline 8 \end{array}$ No need

$L_{90} \propto \frac{1}{Pe^3}$

(4) 1000

4.16 P 30 KN Q 45 KN

$\frac{P}{Q}$ $\frac{L_P}{L_Q} = \frac{(C/Pe)_P^3}{(C/Pe)_Q^3} = \frac{Pe^3_Q}{Pe^3_P} = \left(\frac{45}{30}\right)^3$

$= \frac{27}{8}$ (b)

4.18

$C = 16$ KN

1 MR

$L_{90} = \frac{C}{Pe}$

$= \left(\frac{16 \times 10^3}{2x}\right) = 512$ MR

Reasoning

L, concept

4.20

$P = 2000$ N

$N = 2000$ rpm

$540 = \left(\frac{C}{2}\right)^3 \Rightarrow C = 16.28$ KN

4.1 9800 N

$N = 1000$ rpm

$3000 \times 60 \times 60$

$\begin{array}{r} 18 \\ \times 6 \\ \hline 108 \end{array}$

$108 \times 10^5$

$108 \times 10^5 = \frac{C}{4900N}$

$108 \times 10^3 \times 49 \times 10^2 = C$

$\boxed{108 \times 10^3 + 60^2 \times 1 = C_1}$

SIR

$\because L_{90} \propto \frac{1}{Pe^3}$

$L_{90} = 3000 \times 60 \times 1000$

$= 180$ MR

$Pe = 9800$ N

$Pe' = 4900$ N

$L_{90}' = 8 \times 180$ MR

$8 \times 180 \times 10^6 = h \times 60 \times 2000$

$h = 12000$ hr

v.imp.

Q. A single Row deep groove Ball Bearing has a dynamic load capacity of 40,800 N operates on following work cycle :-

① Radial load of 150,000 N at 500 rpm for 25% of the Time.

② Radial load of 10,000 N at 700 rpm for the 50% of the Time.

③ Radial load of 7,000 N at 400 rpm for the Remaining Time.

calculate the expected half life of Bearing in hours.

Sol

$$P_e = \sqrt[3]{\frac{n_1 P_{e_1}^3 + n_2 P_{e_2}^3 + n_3 P_{e_3}^3}{n_1 + n_2 + n_3}}$$

$$P_e = \sqrt[3]{\frac{500(15\times10^3)^3 + 700(10,000)^3 + 400(7000)^3}{500 + 700 + 400}}$$

$$P_e = \sqrt{\frac{5(15\times1)^3 + 7(1)^3 + 7(7)^3}{16}}$$

$P_e =$

SIR

Time = x min

| Load | Speed | Time | No. of Revolution |
|---|---|---|---|
| Fr = 15000 N $P_{e1}$ | 500 rpm | .25x min | 125x "rev" $n_1$ |
| Fr = 10,000 N $P_{e2}$ | 700 rpm | .5x min | 350x "rev" $n_2$ |
| Fr = 7000 N $P_{e3}$ | 400 rpm | .25x min | 100x "rev" $n_3$ |

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$$P_e = \sqrt[3]{\frac{125x(15000)^3 + 350x(10{,}000)^3 + 100x(7000)^3}{125x + 350x + 100x}}$$

$$P_e = 11192.32\,N$$

$$C = 40{,}500\,N$$

$$L_{90} = \left(\frac{C}{P_e}\right)^3 = \left(\frac{40{,}500}{11192.32}\right)^3$$

$$\underline{L_{90} = 47.38\ MR}$$

$$h \times 60 \times N_{av} = 47.38 \times 10^6$$

$$N_{av} = \frac{\text{Total No. of Revolution}}{\text{Total time}}$$

$$N_{av} = \frac{125x + 350x + 100x}{x}$$

$$\underline{N_{av} = 575\ rpm}$$

$$h \times 60 \times 575 = 47.38 \times 10^6$$

$$h_{(90)} = 1373.3\ hr$$

$$h_{(50)} = 5 \times 1373.3\,hr = \underline{6866.6\ hr}$$

Q A Ball Bearing has anticipated to have a life of 400 MR at a load of 10 kN with 80% Reliability. Find out the :-

(a) Life $L_{30}$ when load is doubled.

(b) find out the life with 60% Reliability under a load of 15 kN.

Sol

$L = 400$ MR

$P = 10$ kN

$R = 80\%$

$L_{30} \propto \left(\frac{1}{Pe}\right)^3$

$L_{30} \quad \left(\frac{1}{2Pe}\right)^3$

8 Pe.

(a) 50 MR

$L_{60} \Rightarrow \quad Pe \to 15000$ N

$L_{90} = \left(\frac{400}{0.8}\right)^{1/1.3}$

24.98

15000

0.10

$\left[Pe \uparrow \to bl\downarrow\right]$

$\ln\left(\frac{500}{2}\right)^3$

S1R $\quad Pe = 10$ kN $\quad L_{90} \propto \frac{1}{Pe^3}$

$L_{80} = 400$ MR

$$\frac{L_{90}}{L_{80}} = \left[\frac{\ln(1/.9)}{\ln(1/.8)}\right]^{1/1.17}$$

$$\boxed{\begin{array}{l} Pe = 10 \text{ kN} \\ L_{90} = 210.62 \text{ MR} \end{array}}$$

$Pe' = 20$ kN

$L_{90}' = \frac{210.62}{8} = 26.32$ MR

$$\frac{L_{30}'}{L_{90}'} = \left[\frac{\ln(1/.7)}{\ln(1/.9)}\right]^{1/1.17}$$

$L_{30}' = 74.65$ MR

$Pe'' = 15$ kN

$L_{90}'' = \frac{210.62}{(1.5)^3} = 62.4$ MR

$$\frac{L_{60}''}{L_{90}''} = \left(\frac{\ln(1/.6)}{\ln(1/.9)}\right)^{1/1.17}$$

$L_{60}'' = 240.5$ MR

# Basic Load Capacity / static load Capacity →

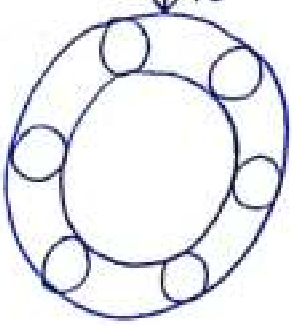
⇓

used for Theoretical Design

OR

static Design

$\downarrow P_e$



$\sigma_{ind} = \frac{P_e}{n \cdot A}$

safe condition

$\sigma_{ind} \le \sigma_{permi}$

$\frac{P_e}{n \cdot A} \le \sigma_{per}$

$\boxed{(P_e)_{max} = n \cdot A \cdot \sigma_{per}}$

⇓

static load Capacity

Static load capacity $\propto$ No. of Rolling element.

Fatigue life is independent of speed.

that fit can fail in 400 rev → No. of cycle

चाहे speed से पहुचो या रुक के

fails

Fatigue life independent of Rolling element, Time

Dynamic load capacity

'C'

⇓

Actual Design

or

Fatigue Design

independent from no. of Rolling element

25/11/2016

# CLUTCH

Clutch is defined as a machine element whose function is to engage and disengage the driver and the driven shaft at the will without (start) stopping the prime-mover.

Clutch is used to avoid frequent stopping & starting of the prime mover but driven machinery can be stop and start frequently.

## *Clutch and Coupling :-

| S.No. | CLUTCH | COUPLING |
|---|---|---|
| 1 | Clutches are used to obtain temporary connection b/w driver and the driven shaft. | coupling are used to obtain permanent connection b/w driver and the driver shaft. |
| 2. | Clutches are used in application where driven machinery required interminent service. | Couplings are used in applications where driven machinery req. permanent service (continuous service). |
| 3. | Clutches are used to transmit power between two collinear shafts only. | couplings are used to transmit power between two collinear as well as non-collinear shaft |

clutches

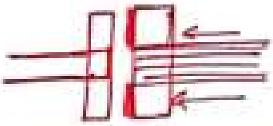
Rigid coupling
- sleeve/Muff coupl.
- flange coupling

- universal hooks joint
- oldham's coupling
- Bush pin coupling

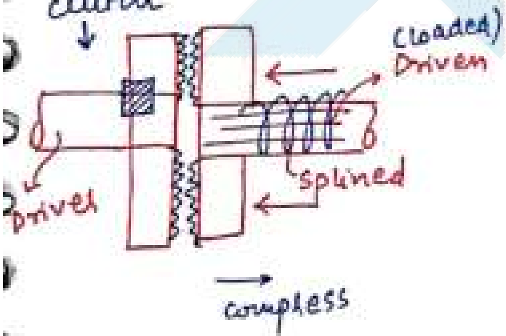
| S.No. | CLUTCH | COUPLING |
|---|---|---|
| 4. | In presence of clutches, the driven machinery can run at variable speed | In presence of coupling, driven machinery can run only speed equal to the driver. |

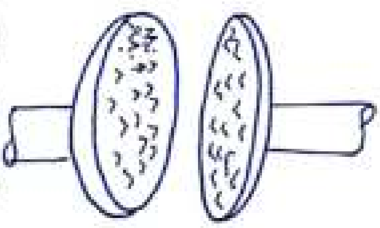
## * CLASSIFICATION OF CLUTCHES :→

① → Clutches



- Positive clutches or Jaw clutches or Claw clutches
  - Saw teeth clutch
  - Square clutch
  - Spiral clutch
- Friction clutches
  - Axial clutches (Force applied for engagement in an axial dirn.)
    - Plate/Disc clutches
      - Single Plate clutch
      - Multi Plate clutch
    - Cone clutches
  - Radial clutch (in a Radial dirn.)
    - centrifugal clutch



## Positive clutches

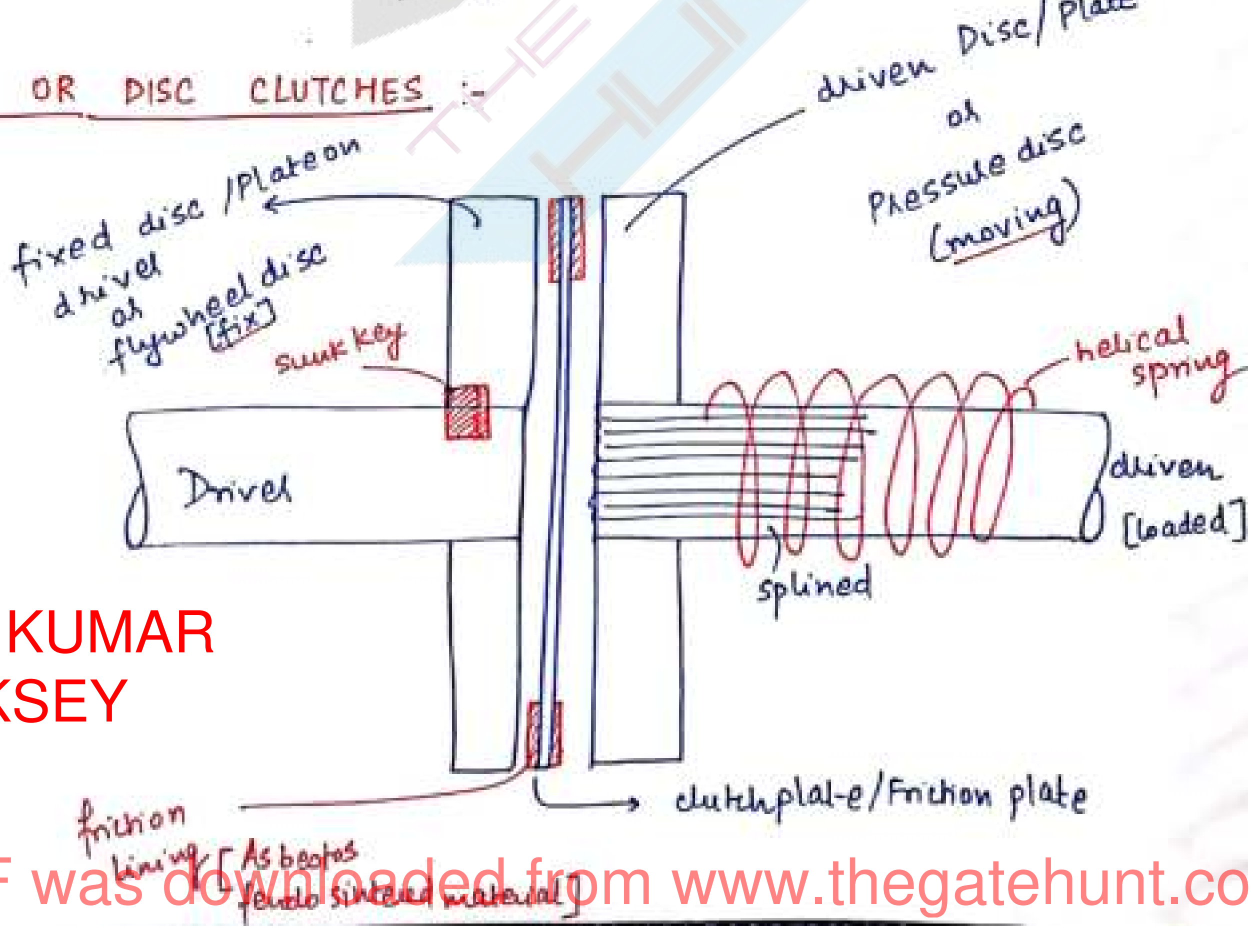
① These are +ve drives, hence no slip occurs.
② Sudden engagement, hence high impact and Jerk.
③ can be limited to low speed application only.
④ clutch can only engaged when unloaded.
⑤ Power transmission capacity more.

Ex:- Rolling mills,
Power presses etc.

## Friction clutches

① These are ~~select~~ slack drives (not -ve drive), hence slip occurs here.
② gradual engagement.
③ less impact and Jerk.
④ can be used at high speed conditions.
⑤ can be engaged when loaded.
⑥ Power transmission capacity less.

# PLATE OR DISC CLUTCHES :-

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$\omega_1$ = angular speed of the driver at the time of engagement Begin.
$\omega_2$ = angular speed of the driven at the time of engagement Begin.
$\theta_1$ = —"— displacement —"— driver —"—
$\theta_2$ = —"— " —"— driven.
$\alpha_1$ = —"— acceleration of the driver.
$\alpha_2$ = —"— —"— driven.
$I_1$ = M.O.I. of the driver.
$I_2$ = M.O.I. of the driven.
$T_1$ = Torque to be transmitted by driver.
$T_2$ = Torque on the driven shaft.

**Driver**

$$T_1 = -I_1\alpha_1$$

$$\alpha_1 = \frac{-T_1}{I_1}$$

$$\frac{d^2\theta_1}{dt^2} = \frac{-T_1}{I_1}$$

$$\frac{d\theta_1}{dt} = -\frac{T_1}{I_1}t + C$$

angular speed of driver at any time

when $t=0 \Rightarrow \frac{d\theta_1}{dt} = \omega_1$

$$C = \omega_1$$

$$\Rightarrow \frac{d\theta_1}{dt} = \frac{-T_1}{I_1}t + \omega_1 \quad \text{— (1)}$$

angular speed of driver at any time

**Driven**

$$T_2 = +I_2\alpha_2$$

$$\frac{d^2\theta_2}{dt^2} = \frac{T_2}{I_2}$$

$$\frac{d\theta_2}{dt} = \frac{T_2}{I_2}t + C'$$

at $t=0 \Rightarrow \frac{d\theta_2}{dt} = \omega_2$

$$C' = \omega_2$$

$$\frac{d\theta_2}{dt} = \frac{T_2}{I_2}t + \omega_2 \quad \text{— (2)}$$

angular speed of driven at any time

t = slip time/engagement completion time/ time to bring driven speed equal to the driver

$$\frac{d\theta_1}{dt} = \frac{d\theta_2}{dt}$$

$$-\frac{T_1}{I_1}t + \omega_1 = \frac{T_2}{I_2}t + \omega_2$$

if $T_1 = T_2 = T$

$$-\frac{T}{I_1}t + \omega_1 = +\frac{T}{I_2}t + \omega_2$$

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$$\boxed{t = \frac{(\omega_1 - \omega_2)\; I_1 I_2}{(I_1 + I_2)\, T}}$$

at Rated speed

$$\omega_{slip} = \left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$$

$$\omega_{slip} = \left[-\frac{T}{I_1}t + \omega_1 - \frac{T}{I_2} - \omega_2\right]$$

T = Torque

rate of energy loss = $T \cdot \omega_{slip}$

$$\underset{(\text{Power loss})}{u} = T\left[-\frac{T}{I_1}t + \omega_1 - \frac{T}{I_2}t - \omega_2\right]$$

$$\int_0^E dE = \int_0^t u \cdot dt$$

$$\boxed{E_{loss} = \int_0^t T\left[-\frac{T}{I_1}t + \omega_1 - \frac{T}{I_2}t - \omega_2\right] dt}$$

Q (ESE 2015) A single plate clutch is designed to transmit 10 kW power at 2000 rpm (not a rated speed). The equivalent mass and radius of gyration of input shaft are 20 kg and 75 mm resp. and the equivalent mass & Radius of gyration for output shaft are 35 kg & 125 mm resp. calc. the time Req. to bring output shaft to the Rated speed from Rest & also find out energy losses during slip time.

$$P = \frac{2\pi NT}{60}$$

$$T = 477.46\ N\text{-}m$$

$$\frac{\omega_1 - \omega_2}{(m_1k_1^2 + m_2k_2^2)\,T} I_1 I_2$$

$$\frac{209.436 - 56.2}{0.112 + 0.546}$$

$$P = \frac{2\pi N T}{60}$$

$$10\times10^3 = \frac{2\times\pi\times2000\times T}{60}$$

$$T = 47.746 \text{ N-m}$$

$$t = \frac{\omega_1 - \omega_2}{(I_1+I_2)T} \quad I_1 I_2$$

$$I_1 = m_1 k_1^2 = (20)(0.075)^2 = 0.1124$$

$$I_2 = m_2 k_2^2 = (35)(0.125)^2 = 0.546$$

$$I_1 = m_1 k_1 = 20\times0.075 = 1.5$$

$$I_2 = m_2 k_2 = 35\times0.125 = 4.37$$

$$\omega_1 = \frac{2\pi\times2000}{60} = 209.43$$

$$t = \frac{(209.43-0)}{(1.5 + 4.37)\,47.746}(1.5\times4.37)$$

$$t = 0.409 \text{ sec}$$

$$E_{loss} = \int_0^{0.409} 47.746\left[-\frac{47.746}{0.112} + 209.43\right]$$

$$E_{loss} = \left[-\frac{T^2}{I_1}\frac{t^2}{2} + \omega_1 t - \frac{T}{I_2}\frac{t^2}{2}\right]_0^{0.409}$$

$$= 2046\text{ J}$$

*Conclusions→

① due to more no. of frictional contact surfaces in multiplate clutch, more heat generated. Hence, coolant required (oil). so there will be a reduction in $\mu$. hence, the power transmission capacity of the multiplate clutch decreases

② In 4 wheelers where radial space is not a constraint, single plate clutch preferred over multiplate clutch.

③ For the given power transmission capacity, multiplate clutch are compact in size. hence in 2 wheelers, ~~single~~ multi plate clutch preferred over single plate clutch.

*********** $\boxed{\text{After Engagement completed}}$ ********

[Single Plate clutch]

UPT (New clutch) | UWT (clutch)

clutch-"Power"→ "Transmission"
↓
friction lining
↓
$T_f$ → axial load

B→W"→[arm support]
↓
collar
↓
$T_f$
↓
Press


**UPT (New clutch)**

① Power $= \frac{2\pi N T_f}{60}$

$T_f$ = Known

③ $T_f = \frac{2}{3}\mu\pi P_{per}(R_o^3 - R_i^3)$

③ Axial load $= \pi(R_o^2 - R_i^2) P_{per}$.

**UWT (clutch)**

① $P = \frac{2\pi N T_f}{60}$

$T_f$ = Known

② $T_f = \mu\pi P_{per} R_i(R_o^2 - R_i^2)$

③ Axial load $= 2\pi R_i(R_o - R_i) P_{per}$

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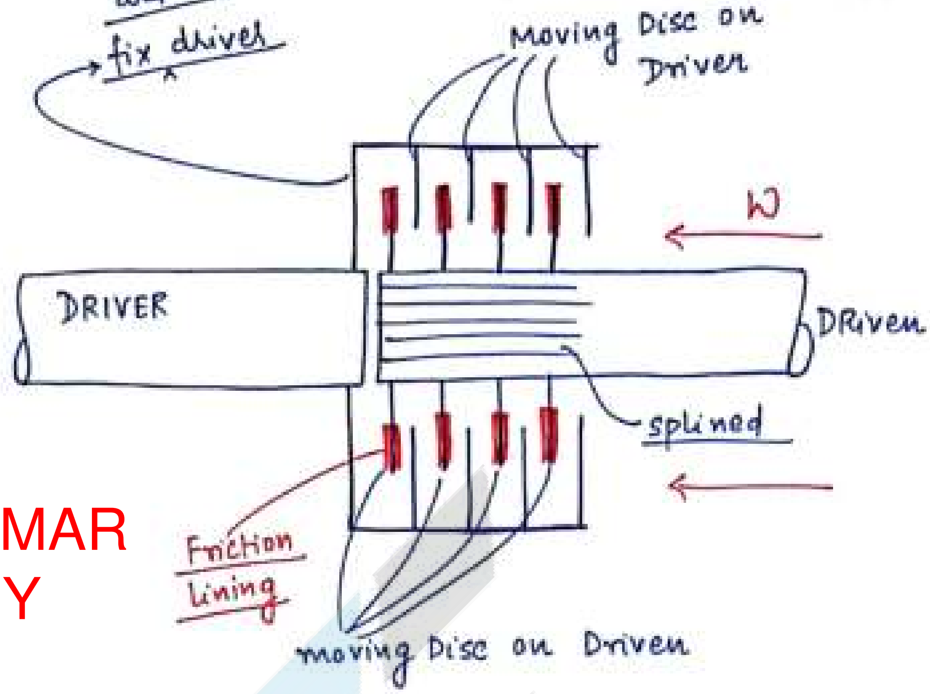
## * MULTIPLATE CLUTCH :-

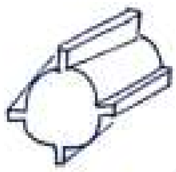
Triple plate clutch

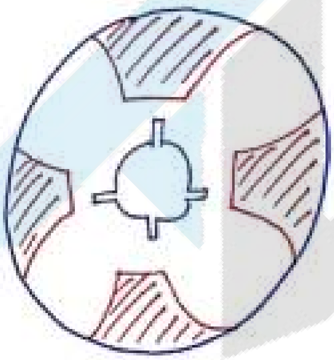
effective on both sides

(n=6)

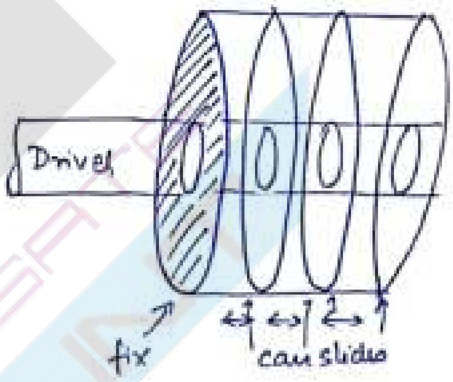




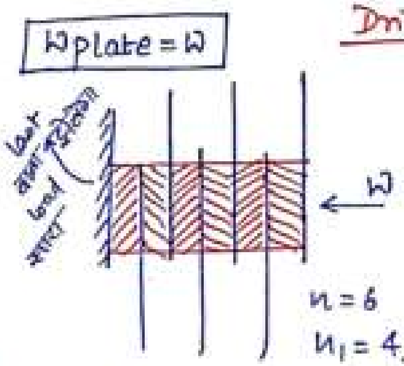





Driven Disc

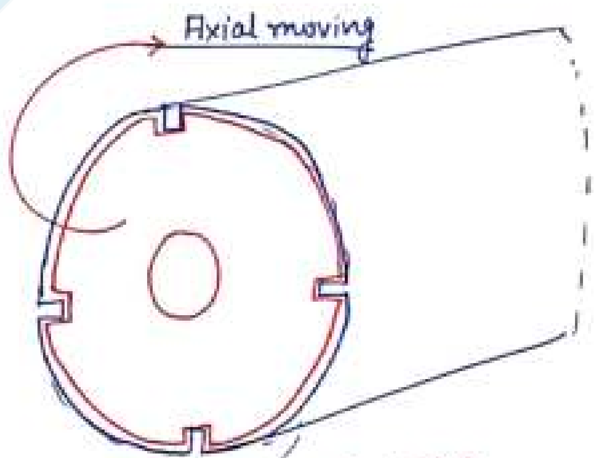



Axial moving




$n = 6$

$n_1 = 4, n_2 = 3$




e.g. $n = 1 \Rightarrow$

$n \rightarrow$ Total no. of friction contact surface

$n_1 \rightarrow$ no. of disc on driver

$n_2 \rightarrow$ " driven

$n = n_1 + n_2 - 1$ → if e.g. $n = 10, 8$, $\cancel{n = 11}$, $n = 12$

always even

$n_1 = n_2 + 1$

$n=1 \rightarrow$ single plate clutch $\rightarrow$ odd (can possible)

$n=2 \rightarrow$ single plate clutch effective on both sides. $\rightarrow$ even

$n=4 \rightarrow$ Double plate clutch effective on both sides. $\rightarrow$ even

## Multiplate clutch

**UPT**

$W_{actual} \rightarrow \frac{W}{n}$

$T_f = n T_{actual}$

$T_f = \left[ n \cdot \frac{2}{3} \mu W_{actual} (\;) \right]$

$T_f = \frac{2}{3} \mu W (\;) ]$

$T_f = n (T_f)_{plate}$

$T_f = n \left( \frac{2}{3} \mu W_{plate} (\;) \right)$

$T_f = n \frac{2}{3} W (\;)$

$\boxed{\because W_{plate} = W}$

① Power $= \frac{2\pi N T_f}{60}$ $\quad T_f =$ Known

③ $T_f = n \cdot \frac{2}{3} \mu \pi P_{per} (R_o^3 - R_i^3)$

③ Axial load $= \pi (R_o^2 - R_i^2) P_{per}$

**UWT**

① Power $= \frac{2\pi N T_f}{60}$

$T_f =$ Known

② $T_f = n \cdot \mu \pi P_{per} R_i (R_o^2 - R_i^2)$

③ Axial load $= 2\pi R_i (R_o - R_i) P_{per}$.

Q A single plate clutch effective on both sides carries an axial thrust of 1500N and the effective Radius of friction surface is 100mm and $\mu = 0.2$. Find the Torque in N-m that can be transmitted.

Sol 1500 N

Theory → UWT

$T_f = n\mu\pi P_{per} R \quad = 60$

$= n\mu W R_{eff} = 60$ Ans

Q A multiplate clutch transmit 50kW power at 1400rpm and the Intensity of pressure cannot exceed 0.5 MPa. The inner radius of the friction lining is 80mm and it is 0.7 times of outer radius. The coeff. of friction b/w the surfaces is 0.12. Determine the no. of disc req. on driver and the driven shaft & also find out axial force req. to transmit power.

Sol Theory → UWT

$R_i = 0.7 R_o$

$0.114\,m = R_o$

$\Rightarrow 50\times10^3 = \dfrac{2\times\pi\times 1400\, T_f}{60}$

$T_f = 341.04\,N\text{-}m$

$T_f = n\mu\pi P_{per} R_i (R_o^2 - R_i^2)$

$341.04 = n\times 0.12\times\pi\times 0.5\times10^6 \; 0.080\left((0.114)^2 - (0.080)^2\right)$

$n = 11.42$

$\boxed{n = 12}$

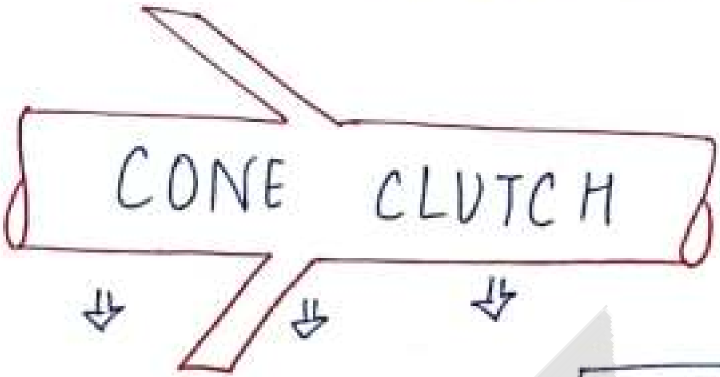
$A.L. = 2\pi R_i (R_o - R_i) P_{per}$

$A.L. = 2563.58\,N$

# "CONE CLUTCH"

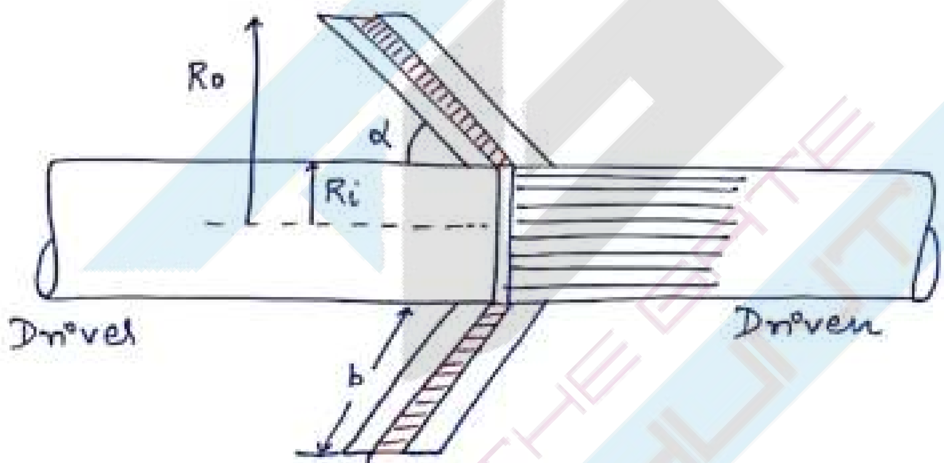
$15^\circ \le 2\alpha \le 30^\circ \rightarrow$ clutches

$120^\circ \le 2\alpha \le 180^\circ \rightarrow$ Bearing




$$\boxed{\mu \rightarrow \frac{\mu}{\sin\alpha}}$$

$\alpha$ = semi cone angle




$$b = \frac{R_o - R_i}{\sin\alpha}$$

Multi cone

**UPT**

① $T_f = n \cdot \frac{2}{3} \frac{\mu}{\sin\alpha} \pi P_{per} (R_o^3 - R_i^3)$

② $W = \pi (R_o^2 - R_i^2) P_{per}$

**UWT**

① $T_f = n \cdot \frac{\mu}{\sin\alpha} \pi P_{per} R_i (R_o^2 - R_i^2)$

② Axial load $= 2\pi R_i (R_o - R_i) P_{per}$

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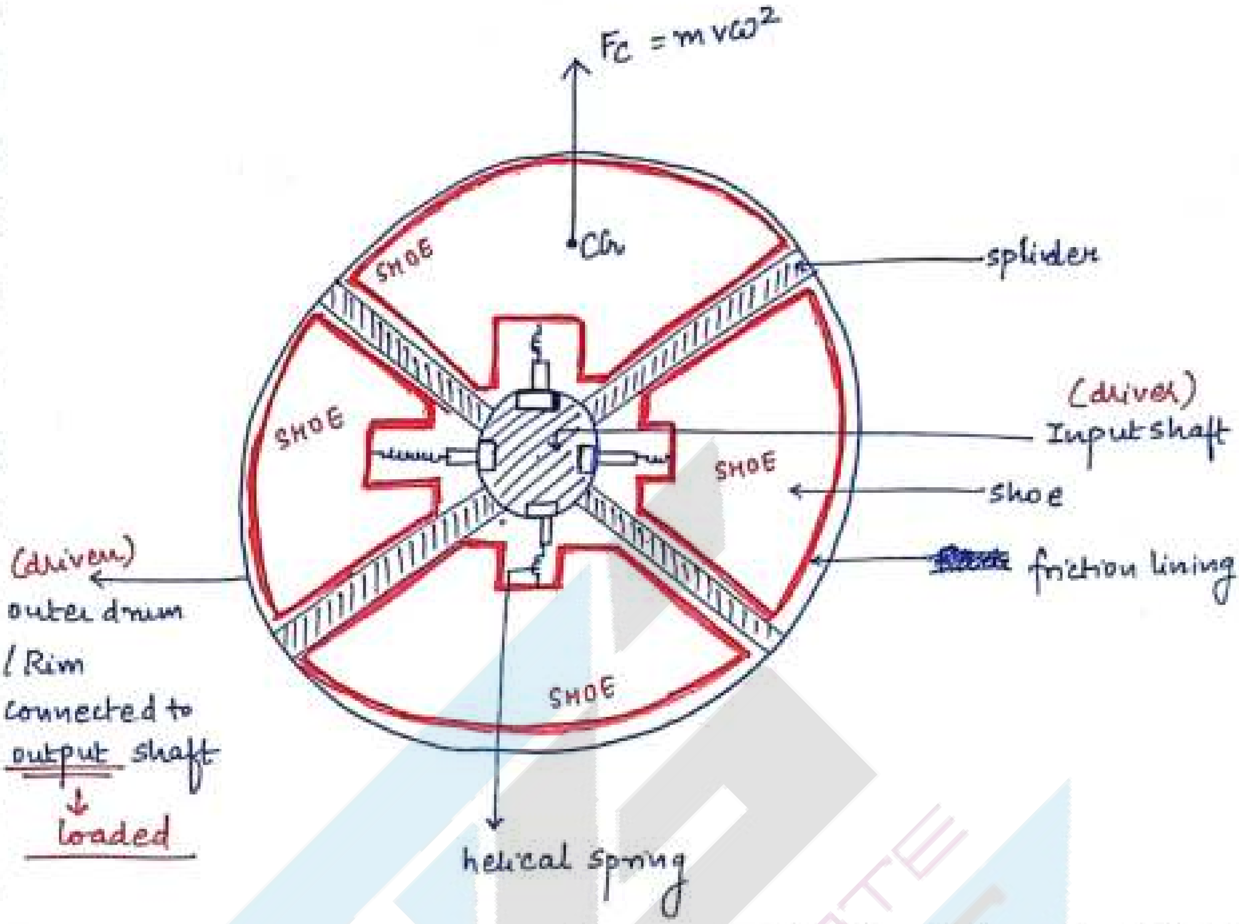
# * Centrifugal Clutch

## (Radial Clutch)



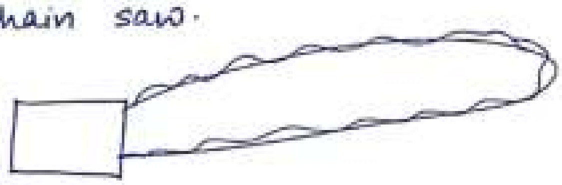

There are some engines, motors, or machinery which produce less power and Torque, so they cannot start with the load, hence, centrifugal clutches are used.

centrifugal clutch permit the engine or motor to start and accelerate to a ~~vert~~ particular velocity without any load, then the load will apply gradually on the driver shaft or input shaft.

e.g. → ① Mopeds.

② Lawn movers.

③ chain saw.



11/12/2016

# BRAKE

- Mechanical Break
  - used for low power absorption
- Electrical Brake
- Hydraulic Brake
- Pneumatic Brake

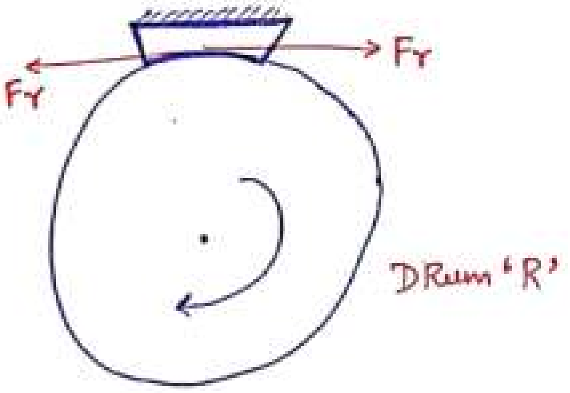
Electrical, Hydraulic, Pneumatic Brake: नहीं पढ़ने हैं → High Power absorption

Mechanical Break (used for low power absorption):

- Radial Brake → Force applied for Breaking in a Radial dirn.
  - Block/Shoe Brake
    - Short-Shoe Brake
    - Long shoe Brake
  - Band Brake
    - Simple Band Brake
    - Differential Band Brake
  - (Band-Block Brake) → No type
- Axial Brake → Force applied for braking in an axial direction.

Brake is defined as a machine element whose function is to retard a moving member, to Bring the moving member into a Stationary condition and to hold the Body at Rest.

Brake perform above function by offering frictional Resistance to a moving member by a stationary member like shoe.



$T_c = F_r \times R$
⇓
Braking torque

Our aim is to find that effort जो level or pedal में लगेगा जिससे की Breaking Torque के लिए

Case-I Drum is a Prime mover

Power = P, N

$$P = \frac{2\pi N T_f}{60}$$

$T_f$ = known

Case-II Drum is not a Prime mover [freely rotating]

$\omega_f = \omega_i + \alpha t$

$\alpha$ = known

$T_f = I \cdot \alpha$
⇓
known

$\frac{10\,m}{\pi D}$ = no. of revolution 'n'

$\theta = 2\pi n$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

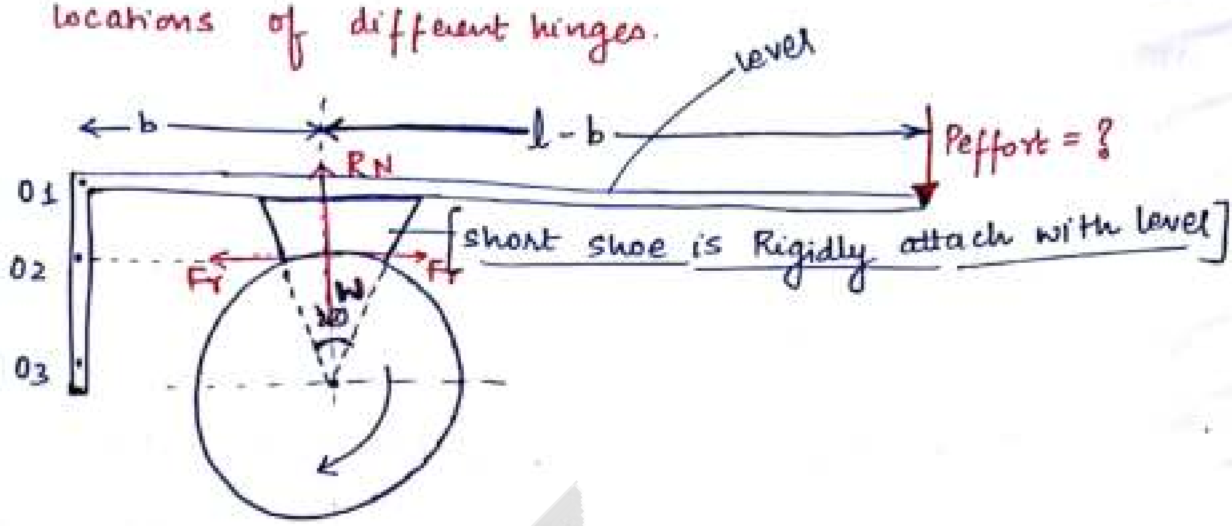
$\alpha$ = known

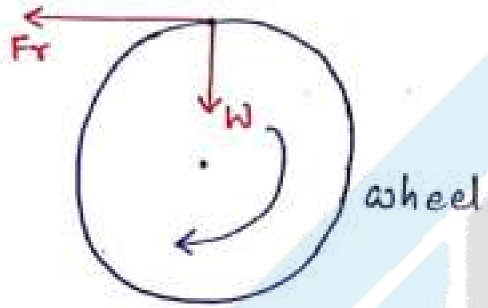
$T_f$ = known

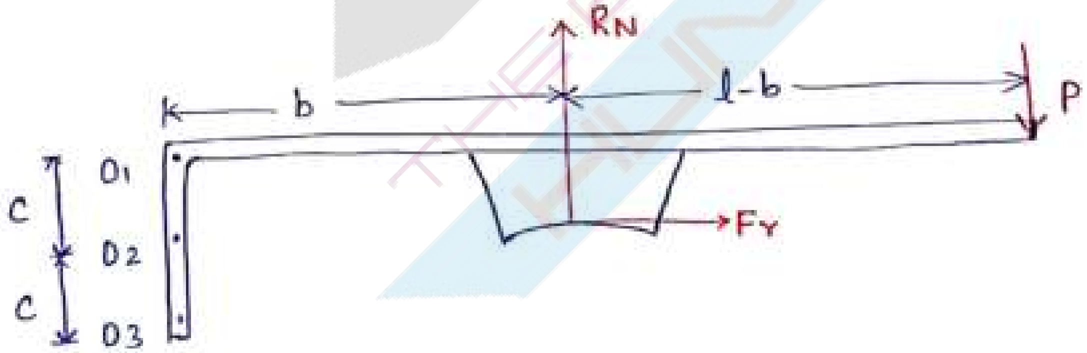
* SHOE BRAKE :-
[short shoe]
$2\theta \le 45°$

N.P.

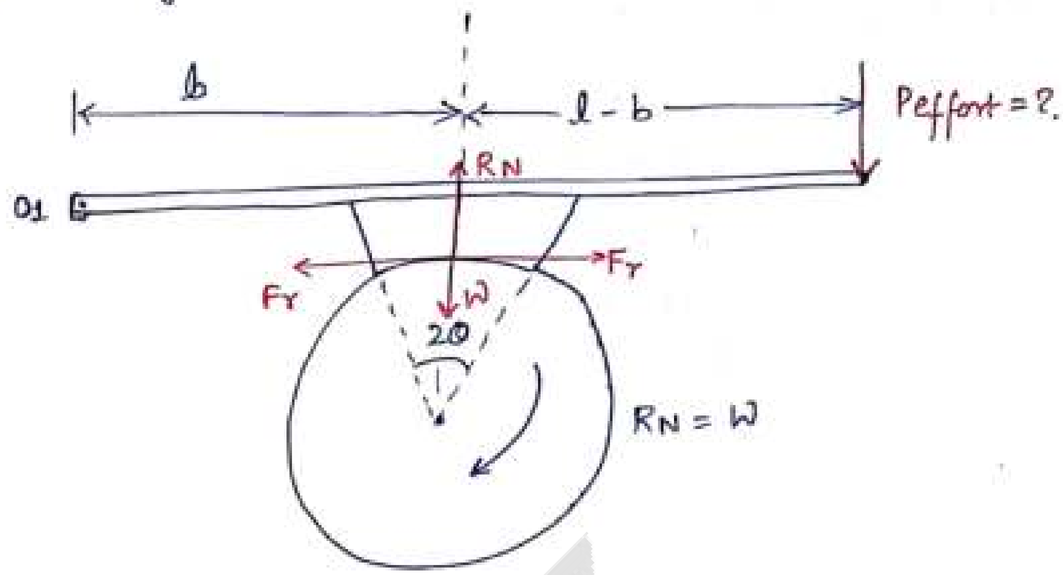
$O_1, O_2, O_3$ are not at a same time.
$\underbrace{O_1, O_2, O_3}_{\text{locations of different hinges.}}$



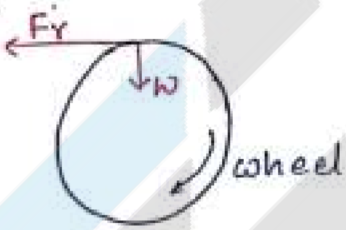



## Case No. 1 (hinge about $O_1$)




**wheel :-**

$T_f$ = known




$F_r \times R = T_f$

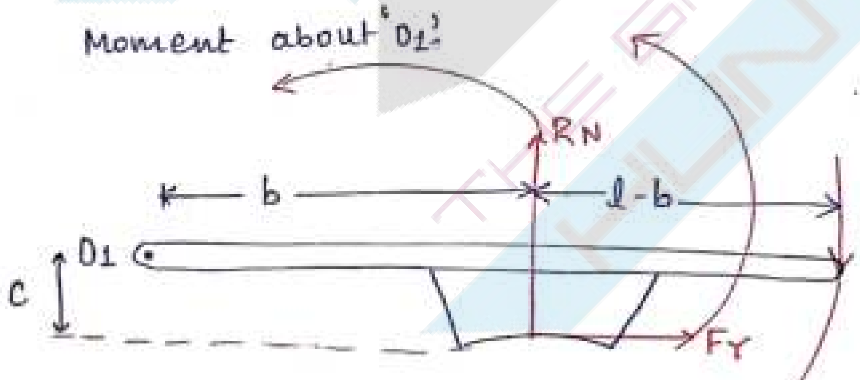
$F_r$ = known

$F_r = \mu R_N$

$R_N$ = known

W = known

**level :-** Moment about '$O_1$'




$$R_N b + F_r c - Pl = 0$$

$$R_N b + \mu R_N c - Pl = 0$$

$$\boxed{P = \frac{R_N (b + \mu c)}{l}}$$

≠0 never

Case No. 2 Hinge about '$O_2$'.

Wheel :- $T_f$ = known

$F_r \times R = T_f$

$F_r$ = known

$F_r = \mu R_N$

$R_N$ = known

$W$ = known

Lever :- Moment about '$O_2$'.


$R_N b + F_r(0) - Pl = 0$

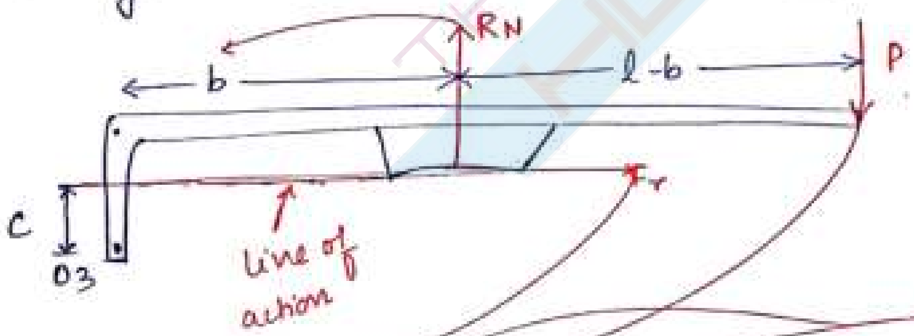
$R_N b + \mu R_N(0) - Pl = 0$

$$\boxed{P = \frac{R_N(b)}{l}}$$

≠ 0 (never)

Case no. 3 Hinge about '$O_3$' — Best when drum rotates 2 (clockwise)

Lever :-




self energizing Brake.

$R_N b - (F_r)(c) - Pl = 0$

$R_N b - \mu R_N(c) - Pl = 0$ — इस बार ≠ 0 ho sakta hai ↓ Next page

$$\boxed{P = \frac{R_N(b - \mu c)}{l}}$$

minm. effort in case 3

if $b = \mu c \Rightarrow P_{effort} = 0 \Rightarrow$ self locking or self Braking

Total undesirable But only in case → screw Jack.

if $b > \mu c \Rightarrow P_{effort} = +ve \Rightarrow$ Controllable Braking

$b = \mu c \Rightarrow P_{effort} = 0 \Rightarrow$ self locking.

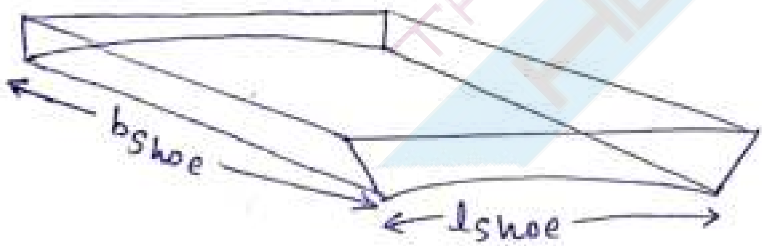
$b < \mu c \Rightarrow P_{effort} = -ve \Rightarrow$ known as uncontrollable Braking.

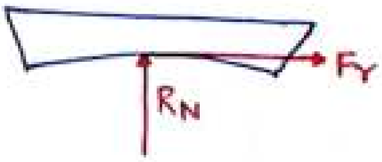
Conclusion → ① Brake is said to be a self Energizing Brake when moment due to friction act in same direction as moment due to effort.

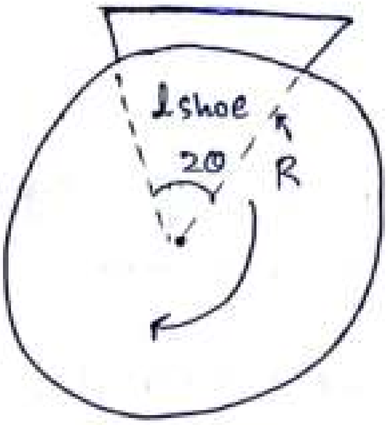
② A self energizing Brake should be designed in such a way that the Brake should not give self locking and uncontrollable Breaking

③ for the given configuration, when drum rotates in clockwise dirn, the fulcrum $O_3$ is the best fulcrum because it gives self energizing Brake.

* DESIGN OF SHOE :-







$$P_{ind} = \frac{R_N}{bl} \qquad l_{shoe} = (2\theta R)$$

$$P_{ind} = \frac{R_N}{b(2\theta)R}$$

Short Shoe

( ∵ Pressure Uniform).

<u>safe condn.</u>

$$P_{ind} \le P_{per}.$$

$$\frac{R_N}{b(2\theta)R} \le P_{per}.$$

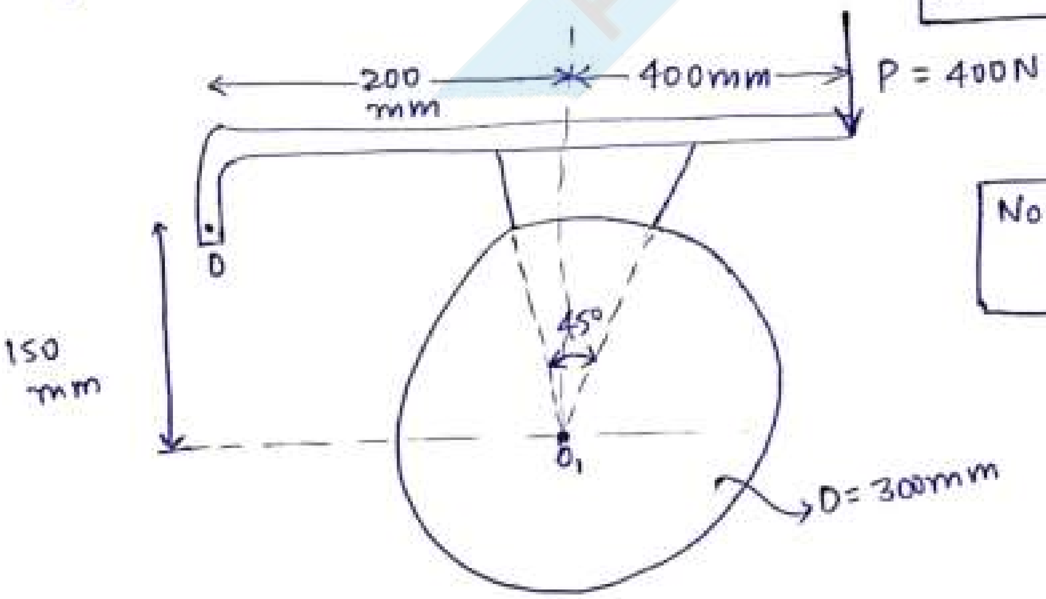
$$\boxed{(R_N)_{max} = b(2\theta)R \cdot P_{per}}$$

⇓

strength of shoe

<u>Q</u>

$\boxed{\mu = .3}$




No other data is given

$T_f = ?$

Sol

$R_N(.200) + \cancel{(.3)R_N(0)} - (400)(600) = 0$

$R_N = \frac{\cancel{240000}}{\cancel{200}}$ 1200

$R_N = ?$

$F_r = \mu R_N = .3 \times 1200$

$= 3600$

$= 360.0$

$F_r \times R = T_f$

$R =$

$F_r \times R = 54.0$

$360.0 \times .150$

360
15

36
×15
180
36
540

SIR

Moment about 'O'

$R_N(200) = 400(600)$

$R_N = 1200N$

$F_f = \mu R_N = 360N$

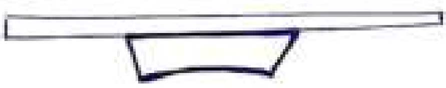
$T_f = 360 \times .15$

$= 54 N\text{-}m$

---

Short shoe is

Rigidly attach with Level



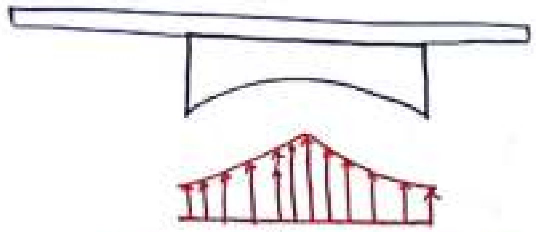
$P_{ind} \Rightarrow$ ↑↑↑↑↑↑↑

Uniform Pressure induced

## * LONG SHOE

[2θ > 45°]

long shoe when Rigidly attach



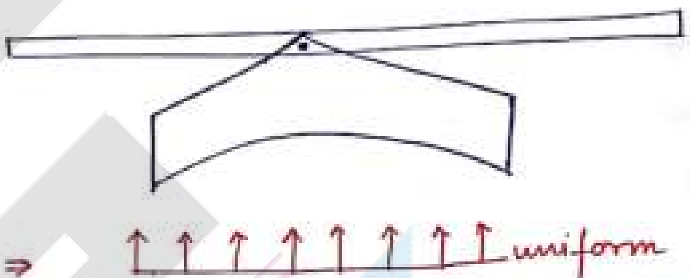
Non - uniform Pressure distribution

Problem

more wear

Solution

long shoe is Pivoted with the level to minimize wear.



Rnd ⇒ uniform distribution

~~$F_f = \mu R_N$~~

(θ → Radian में होता है)

(sin)

sin θ°, sin θ$^c$ > same ans.

But separately differ.

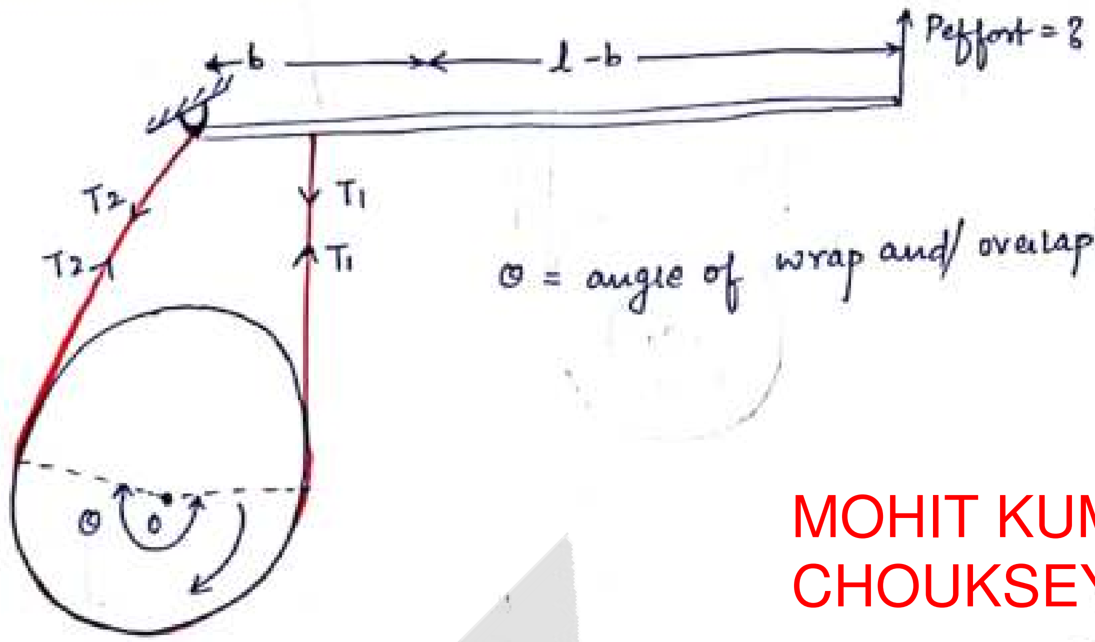
$$\mu \longrightarrow \mu_{eq} = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

long shoe → short shoe

absorbs more Power

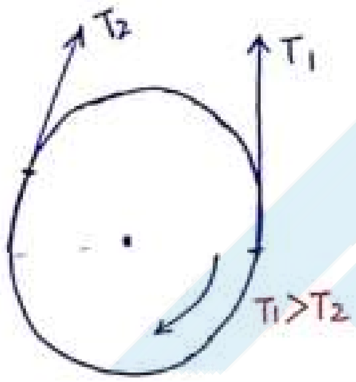
$$\boxed{F_f = \mu_{eq} \cdot R_N}$$

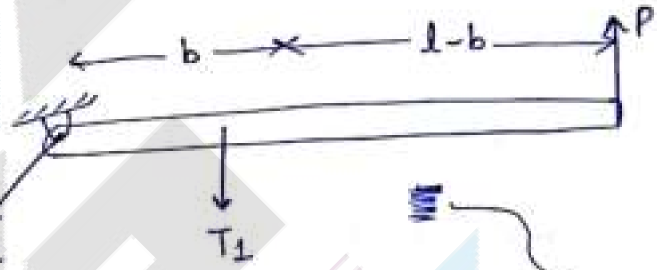
## * BAND BRAKE :- Case No. 1 → Simple Band Brake.




θ = angle of wrap and/ overlap/contact






wheel

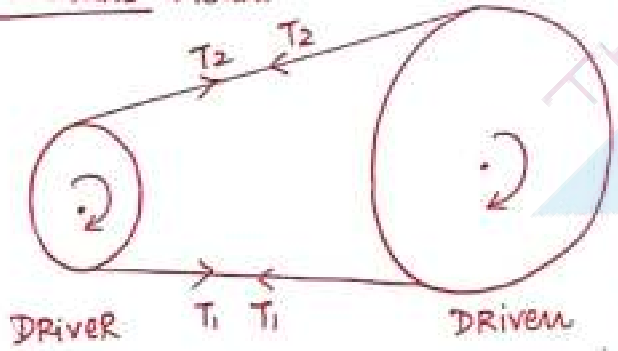
$$T_f = (T_1 - T_2)R \quad \text{—— (1)}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{—— (2)}$$

large Tension → $T_1$; Small Tension → $T_2$

$T_1, T_2$ are known.

mistake → learn

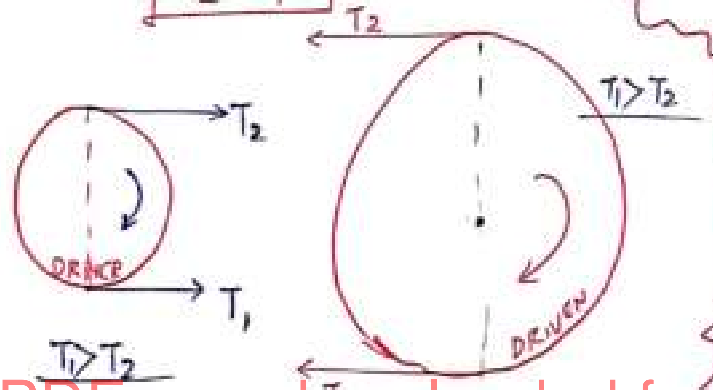



moment about 'O'

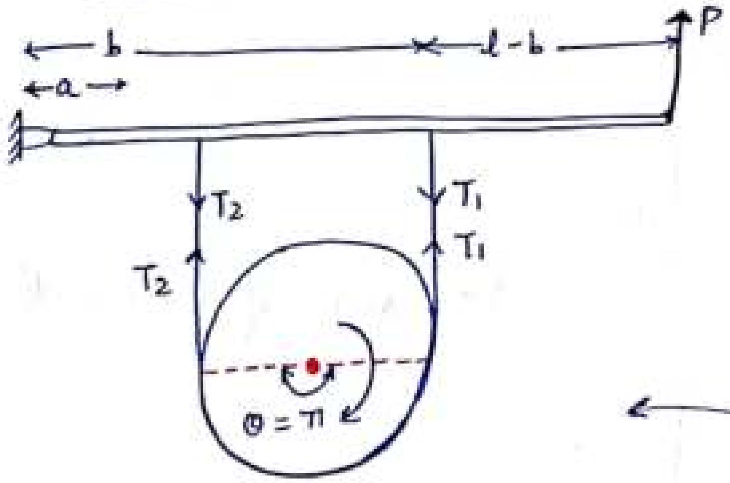
$$T_1 b - Pl = 0$$

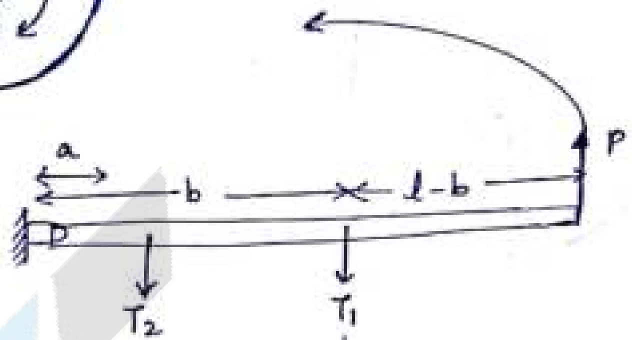
$$\boxed{P = \frac{T_1 b}{l}}$$

$\boxed{T_2 < T_1}$



## * Differential Band Brake ← Case No. 2




**Wheel**

$\tau_f = (T_1 - T_2)R$ —(1)

large
$\frac{T_1}{T_2} = e^{\mu\theta} = e^{\mu(\pi)}$
small

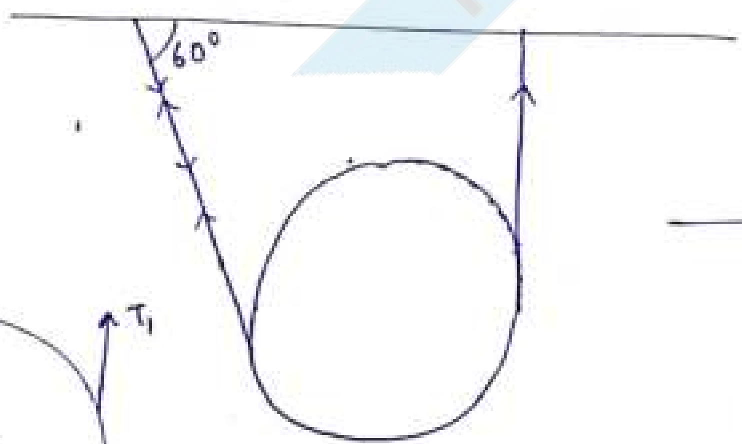
$T_1 > T_2$ ✓

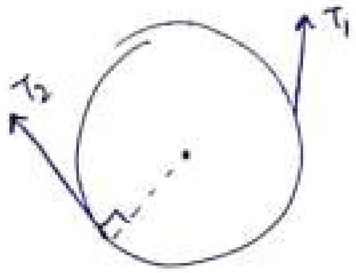
moment about 'O'

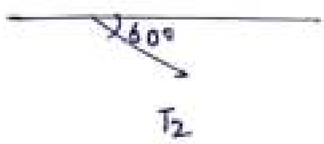
$T_2 a + T_1 b - Pl = 0$

$$\boxed{P = \frac{T_1 b + T_2 a}{l}}$$

*

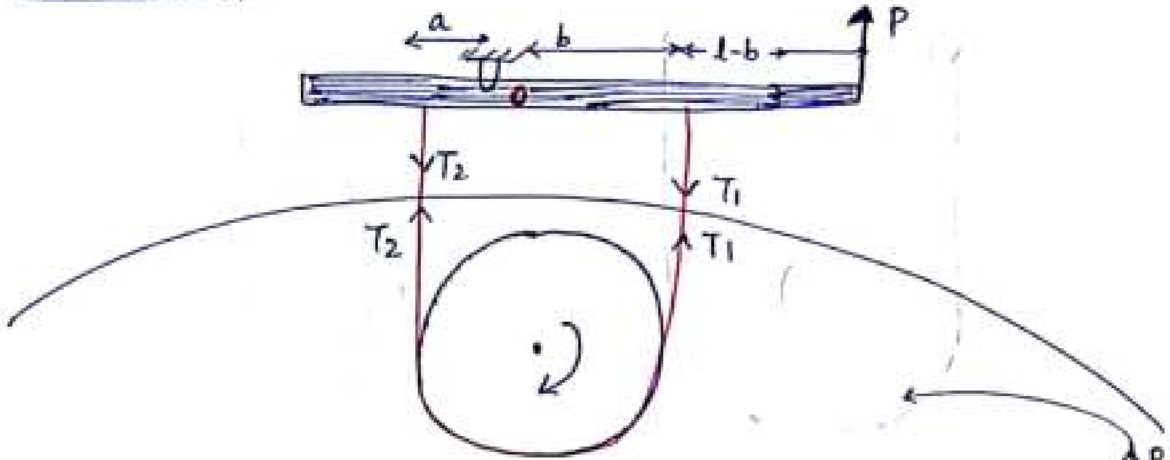


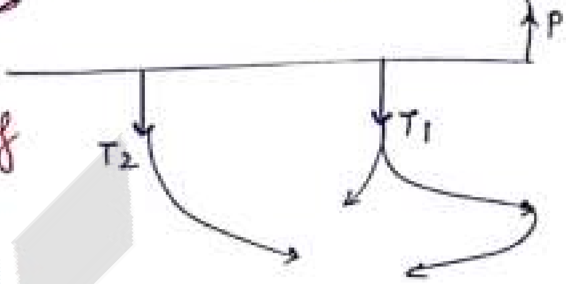




(Resolve them)

Case No. 3 :-




Lever : 'O' moment




self energizing Brake

$T_1 b - T_2 a - Pl = 0$

$$\boxed{P = \frac{T_1 b - T_2 a}{l}}$$

if $T_1 b > T_2 a \longrightarrow P > 0$ – (controllable Braking)

$T_1 b = T_2 a \longrightarrow P = 0 \longrightarrow$ self locking

$T_1 b < T_2 a \longrightarrow P < 0 \longrightarrow$ uncontrollable Braking

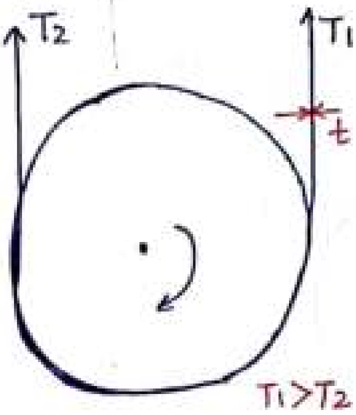
$\Rightarrow P = -ve$

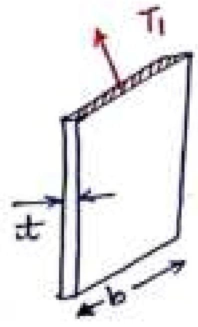
Conclusion :- ①

②

③ Case No. 3 is the Best Brake either the wheel rotates in clock or rotates in anticlock dirn. because it gives self energising Brake.

## * DESIGN OF BAND:-




$$(\sigma_{ind})_{max} = \frac{T_1}{bt}$$

Safe condition

$$(\sigma_{ind})_{max} \le \sigma_{pl}$$

$$\frac{T_1}{bt} \le \sigma_{pel}$$

Tensile strength of Band

$$\boxed{T_{max} = bt\,\sigma_{pel}}$$

Brake completed.

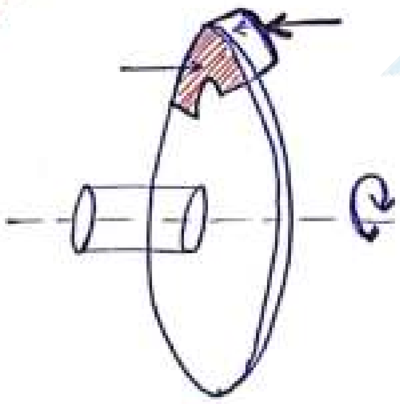
Specials :- Disc Brake

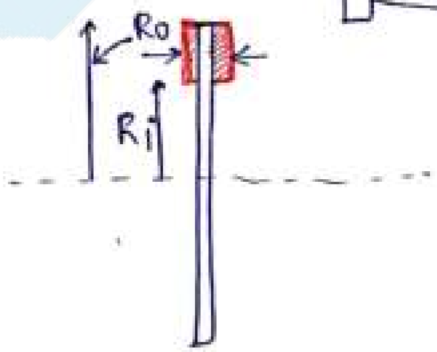
never sefemfized

$$\omega_i \xrightarrow{\text{Brake}} \omega_f$$

$$P_{loss} = T_f \cdot \omega_{mean}$$

$$\boxed{\omega_{mean} = \frac{\omega_f + \omega_i}{2}}$$






UPT

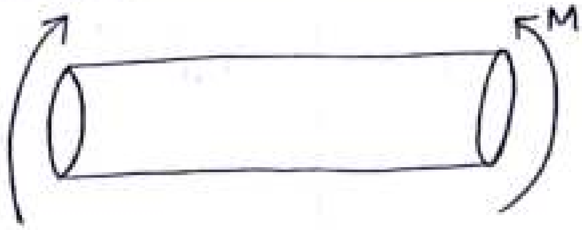
$$T_f = \frac{2}{3} n \cdot \mu \pi P_{per} (R_o^3 - R_i^3)$$

UWT

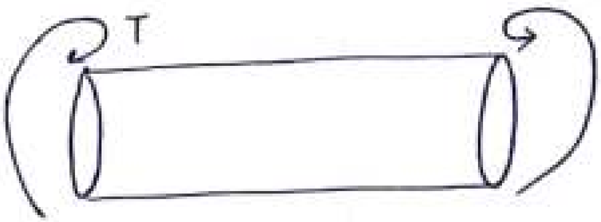
$$T_f = n \cdot \pi P_{per} R_i (R_o^2 - R_i^2)$$

NEW CHAPTER

## FATIGUE DESIGN OF SHAFT



$M = \begin{matrix} +400 \\ -200 \end{matrix}$ kN-mm



$T = \begin{matrix} +300 \\ -100 \end{matrix}$ kN-mm

$P = \begin{matrix} +100 \text{ kN} \\ -20 \text{ kN} \end{matrix}$

MSST

MDT

$$\frac{32M}{\pi d^3} \le \sigma_{per}$$

$$\frac{16T}{\pi d^3} \le \tau_{per}$$

$$\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} \le \frac{S_{yt}}{N}$$

$$\frac{4P}{\pi d^2} \le \sigma_{per}$$

Now, chapter New starts →

**Soderberg's equation**

$$\frac{\sigma_m k_f}{S_{yt}} + \frac{\sigma_a . k_f}{S_{ut}} \le 1/N$$

**Goodman's eqn.**

$$\frac{\sigma_m k_t}{S_{ut}} + \frac{\sigma_a \cdot k_f}{\sigma_e} \le 1/N$$

Soderberg's eqn.
(Ductile)

$$\frac{\sigma_m k_t}{S_{yt}} + \frac{\sigma_a \cdot k_f}{\sigma_e} \le \frac{1}{N}$$

(or)

$$\frac{\tau_m k_t}{S_{ys}} + \frac{\tau_a}{\tau_e} \cdot k_f \le \frac{1}{N}$$

Goodman's eqn.
(Brittle)

$$\frac{\sigma_m}{S_{ut}} k_t + \frac{\sigma_a}{\sigma_e} k_f \le 1/N$$

$S_{yc} > S_{yt} > S_{ys}$ - Ductile

$S_{yc} > S_{ys} > S_{yt}$ - Brittle

$k_t \rightarrow$ Theoretical / static stress concentration factor.

$k_f \rightarrow$ Actual / Fatigue stress concentration factor.

$\sigma_m, \tau_m \rightarrow$ mean stress.

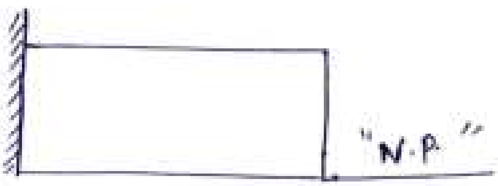
$\sigma_a, \tau_a \rightarrow$ stress amplitude or variable stress.

$N$ = FOS $\rightarrow$ Factor of safety

$\sigma_e, \tau_e \rightarrow$ surface Endurance limit of the machine component under actual working condition.

* $k_t$ (Theo. / Static stress Concn. factor) :-

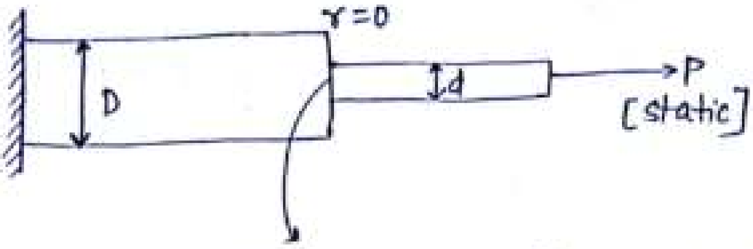
Theoretical $\longrightarrow$ static loading



Theoretical → static loading

$K_{t_1} > 1$

$\sigma_{act} = K_{t_1} \cdot \sigma_0$

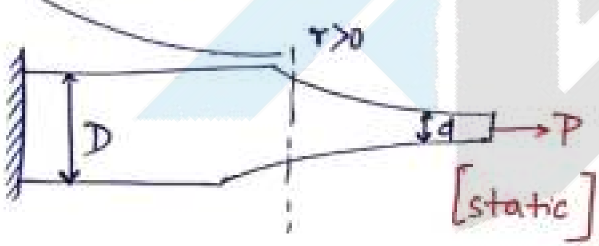



$\sigma_0$ [Nominal stress ⇓ maxm. stress] $= \dfrac{P}{A_{(min)}}$ ; $\sigma_0 = \dfrac{P}{\frac{\pi}{4} d^2}$

$K_{t_2} > 1$

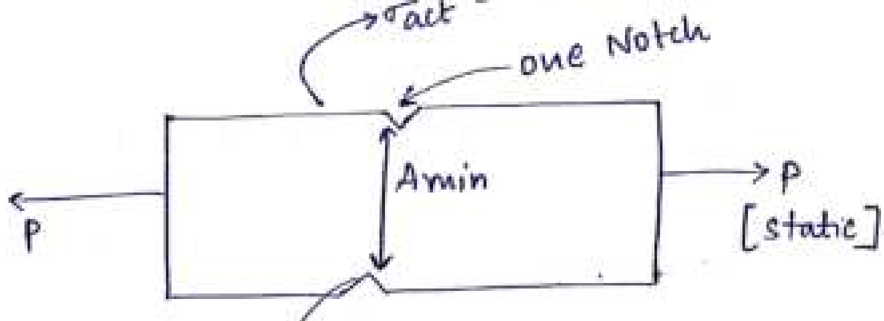
$K_{t_2} < K_{t_1}$

$\sigma_{act} = K_{t_2} \sigma_0$



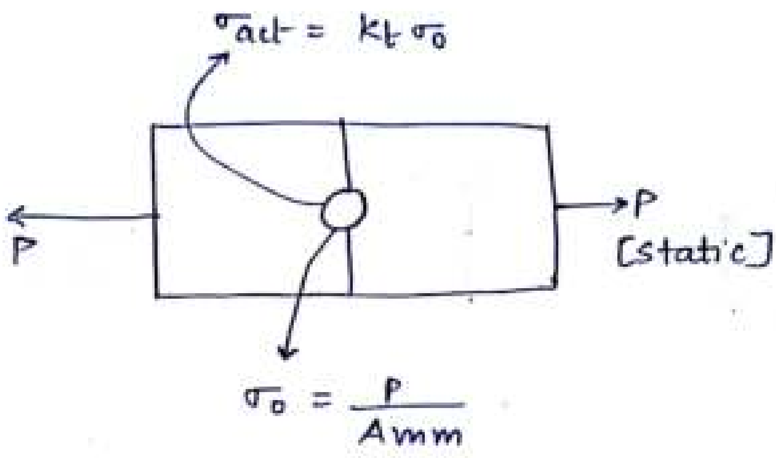

$\sigma_0 = \dfrac{P}{A_{min}} = \dfrac{P}{\frac{\pi}{4} d^2}$

$\sigma_{act} = K_t \cdot \sigma_0 \quad (K_t > 1)$




$\sigma_0 = P/A_{min}$

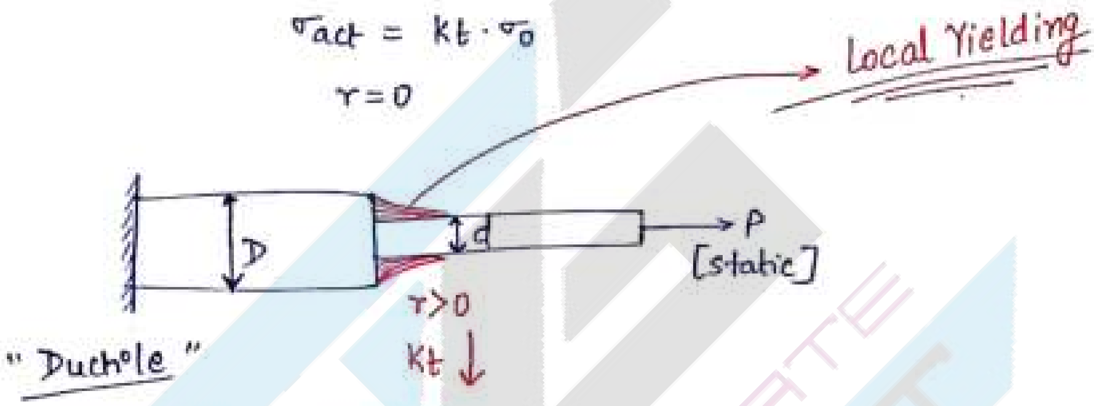
$\boxed{K_t = 3}$




$K_t \uparrow\uparrow\uparrow$

$\sigma_{act} = K_t \cdot \sigma_0$

$r = 0$




$r = 0$

$K_t \uparrow \xrightarrow{\text{local yielding}} K_t \downarrow$, $r > 0$

**Conclusion :-** ① The effect of stress concentration factor under static loading is not serious for ductile material because the geometry near the discontinuity will be rearranged by the phenomenon local yielding, hence $K_t$ can be neglected for ductile ($K_t = 1$).

② The effect of stress concn. factor under static loading is more serious for Brittle material because they doesn't permit any yielding, hence $K_t$ cannot be neglected for Brittle.

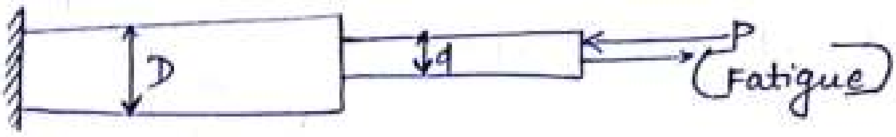
## * Actual/ Fatigue stress concentration factor:- ($K_f$)

$K_f \uparrow\uparrow$

$K_f \cdot \sigma_0$

$\sigma_{act} = K_f \cdot \sigma_0$

$r = 0$




"Ductile"

$\boxed{\text{Fatigue does not Permit any yielding}}$

$\boxed{\text{fatigue fail} \longrightarrow \text{fracture (crack)}}$

The effect of stress concn. factor under fatigue loading is more serious for both the material; ductile and Brittle. Hence, $K_f$ cannot be neglected for any material.

## * $\sigma_m / \tau_m$ (mean stress):-

[with Sign]

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2}$$

Ex:- sign

## * Stress Amplitude / Variable stress ($\sigma_a$) :-

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2}$$

* Stress Ratio = $\frac{\sigma_{min}}{\sigma_{max}}$

* " $\sigma_e$ " ["Surface endurance limit of the machine component under actual working condition".]

"Lab"

For any material under std. condn."

① std ~~condition~~ size

② std surface finish

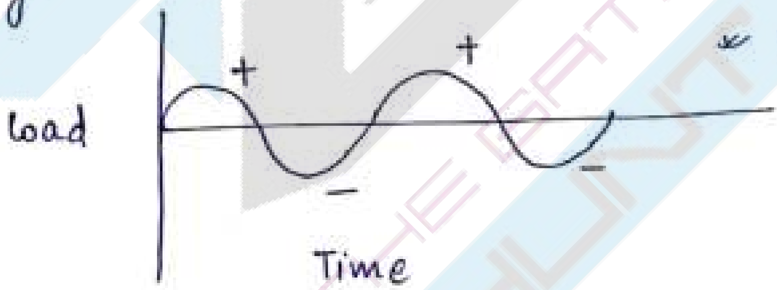
③ std load

$\sigma_e^*$ = std. Endurance limit of material.

* Not a property of material
* Depends upon lab condition


Pure Bending

- $\sigma_{e\,MS}^* = .5\,S_{ut}$
- $\sigma_{e\,CI}^* = .4\,S_{ut}$
- $\sigma_{e\,cast\,alloys}^* = .3\,S_{ut}$




$K_a, K_b, K_c < 1$

$\sigma_e = K_a \cdot K_b \cdot K_c * \sigma_e^*$

$K_a \rightarrow$ size factor $\Rightarrow$ size $\uparrow \rightarrow K_a \downarrow \rightarrow \sigma_e \downarrow$

$K_b \rightarrow$ surface finish factor $\Rightarrow$ Roughness $\uparrow \rightarrow K_b \downarrow \rightarrow \sigma_e \downarrow$

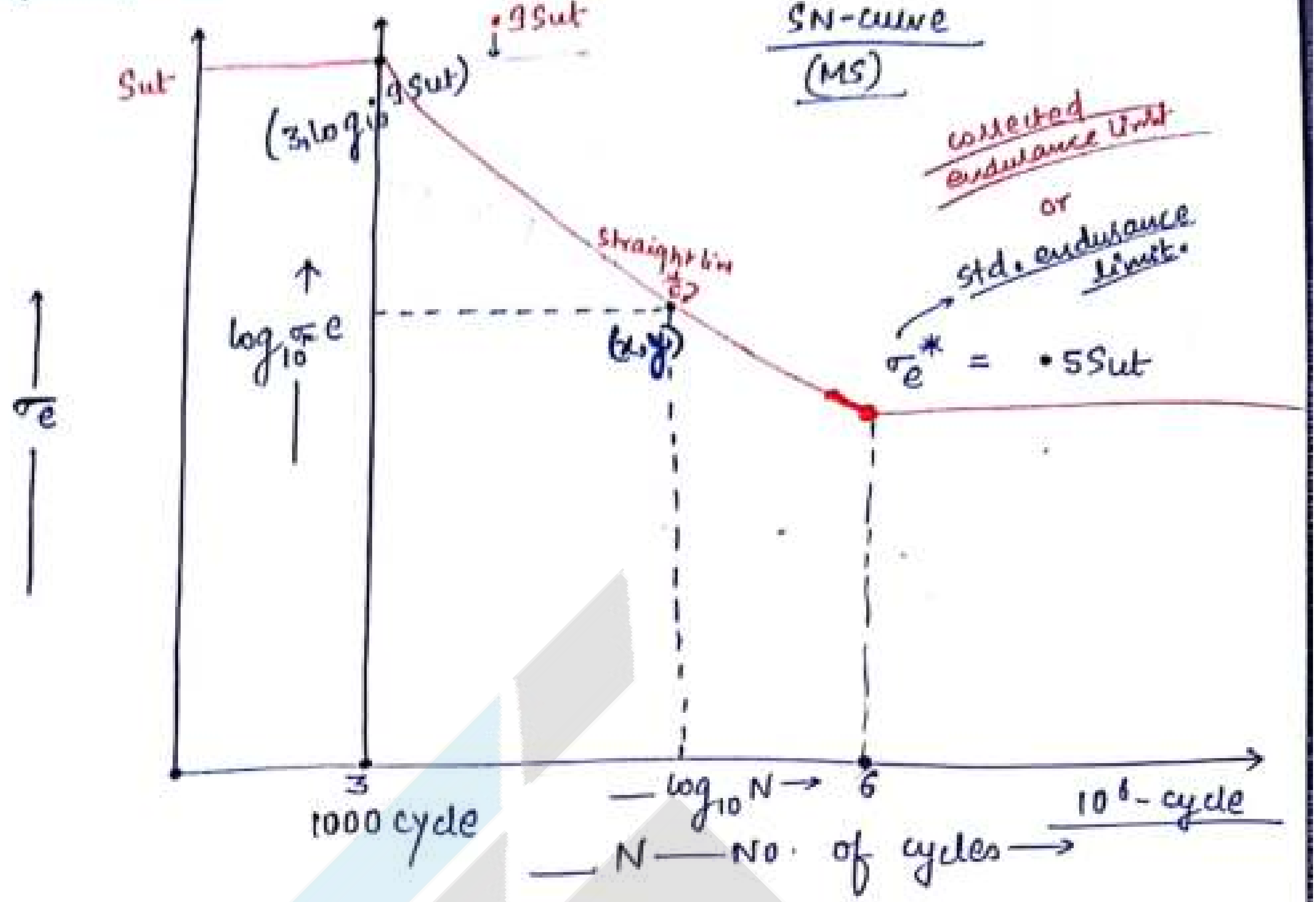
$K_c \rightarrow$ load factor

for Bending $\rightarrow K_c = 1$

Twisting $\rightarrow K_c = .56$

Fixed cost $\rightarrow K_c = .75$

fatigue fail → fracture (crack)




$$\boxed{\text{str. line eqn.} \rightarrow \left(y-\log_{10}.9\,Sut\right) = \frac{\log_{10}.5\,Sut - \log_{10}.9\,Sut}{6-3}(x-3)}$$

Soderberg line → $\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{\sigma_e} \leq \frac{1}{N}$

dia ↑ → Area ↑ → $\sigma_m \downarrow, \sigma_a \downarrow$

Goodman line → $\frac{\sigma_m}{\cancel{S_{yt}}\,S_{ut}} + \frac{\sigma_a}{\sigma_e} \leq 1/N$

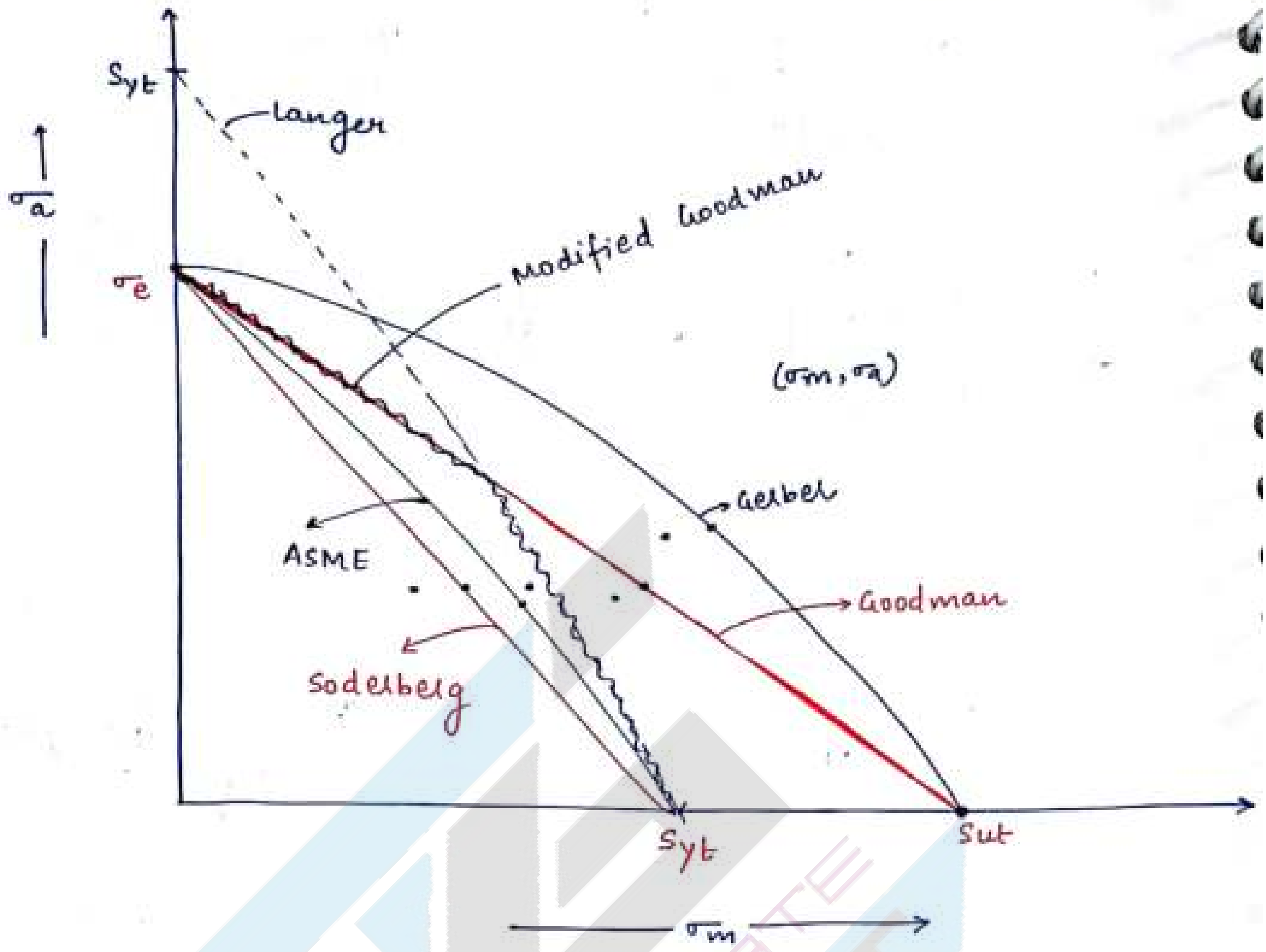
Langer line → $\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \leq 1/N$

Gerber's Parabola → $\left(\frac{\sigma_m N}{S_{ut}}\right)^2 + \frac{\sigma_a N}{\sigma_e} \leq 1$

ASME Ellipsed $\left(\frac{\sigma_m}{S_{yt}}\right)^2 + \left(\frac{\sigma_a}{\sigma_e}\right)^2 \leq 1/N^2$

MOHIT KUMAR CHOUKSEY

$dia \uparrow \rightarrow Area \uparrow \rightarrow \sigma_m \downarrow, \sigma_a \downarrow$



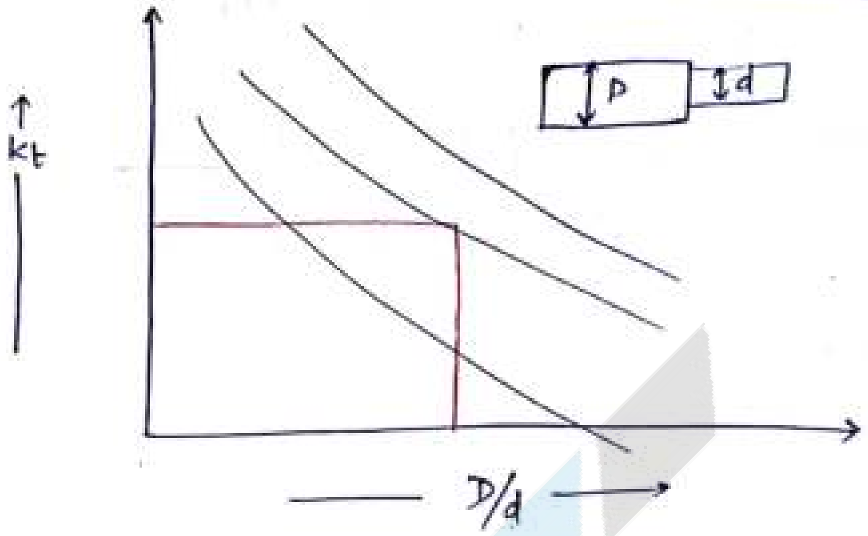

Modified Good ⇓ safe Result of

$$\left[\begin{array}{c} \dfrac{\sigma_m}{S_{ut}} + \dfrac{\sigma_a}{\sigma_e} \le 1/N \\ + \\ \dfrac{\sigma_m}{S_{yt}} + \dfrac{\sigma_a}{S_{yt}} \le 1/N \end{array}\right]$$

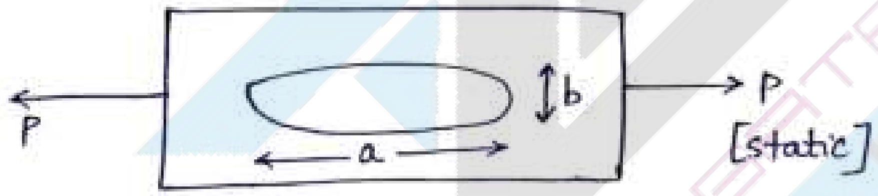
Hence, Soderberg is the most conservative theory.

Find out $k_t = ?$ → chart method [Experimental method 99%]

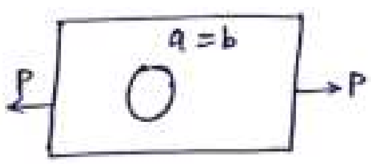
→ formula method



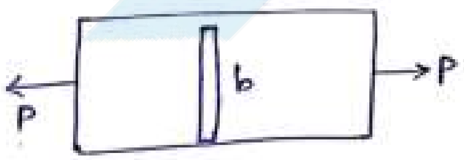

$*$ FORMULA METHOD →




$$\boxed{k_t = 1 + \frac{2b}{a}}$$

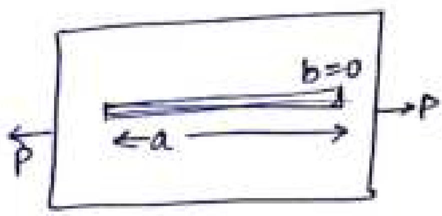



$k_t = 3$



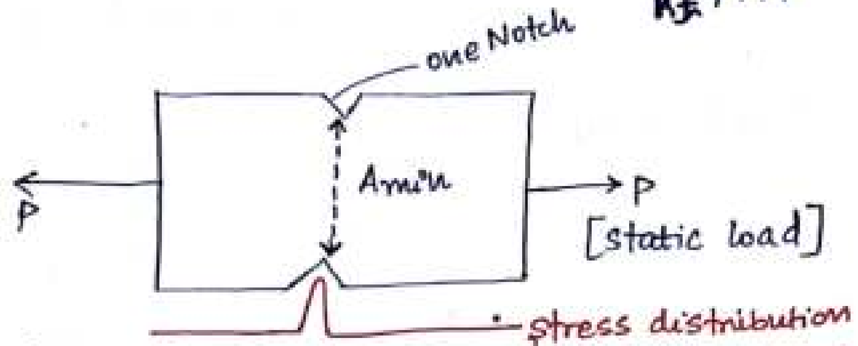

$a = 0$

$k_t = \infty$




$k_t = 1$

$k_t \uparrow\uparrow\uparrow\uparrow$



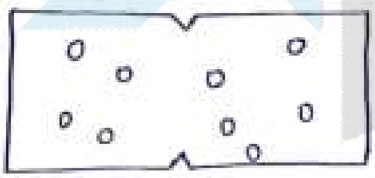

One Notch is very dangerous under static loading for axial load.

## Method Reducing for $k_t \downarrow$

① 

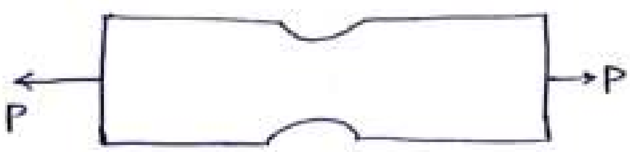

$k_t \downarrow$

② Drilling holes

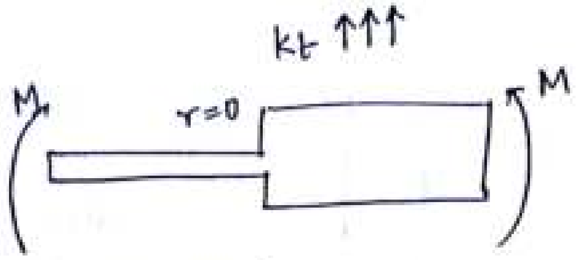


$k_t \downarrow$

③ Removal of material

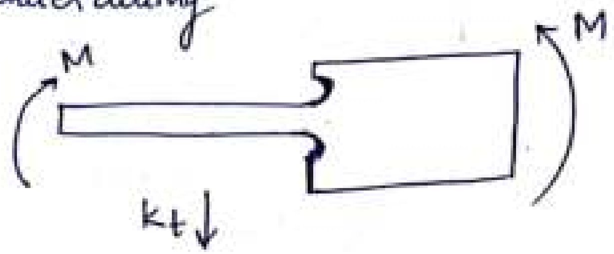



$k_t \downarrow$

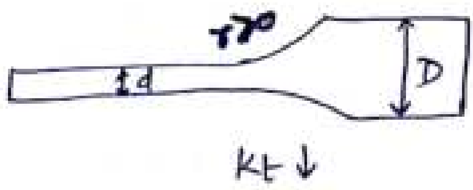



## Method Reducing for kt ↓

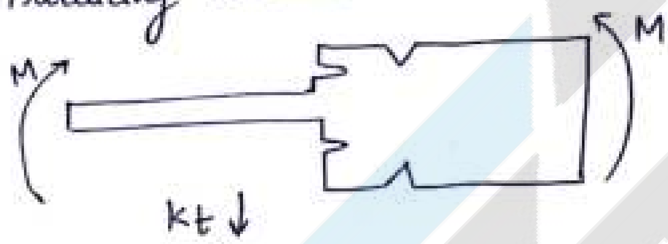
① Under cutting

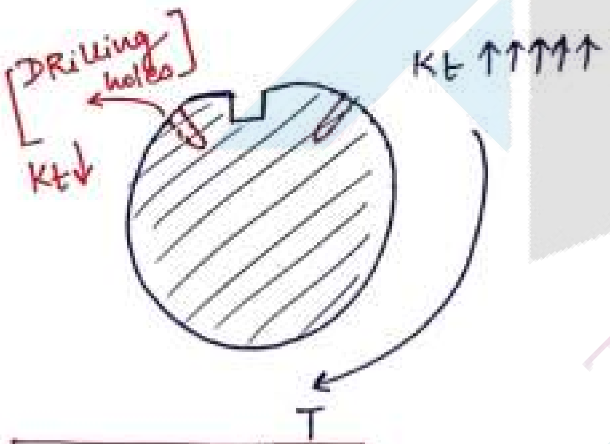



(iii) Provide Radius of curvature




② Producing Notches



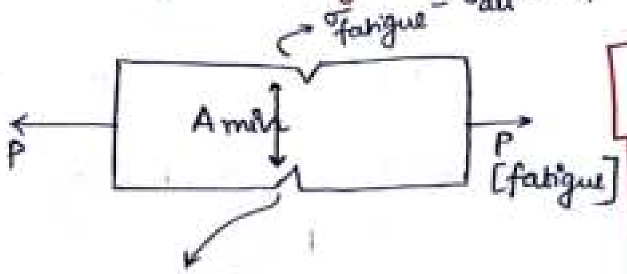

$$\boxed{SE\uparrow = \frac{\sigma^2\uparrow}{2E}}$$

## * Notch Sensitivity :-

**Fatigue loading**

$K_f \uparrow$

$\sigma_{fatigue} = \sigma_{act} = K_f \cdot \sigma_0$




$\sigma_0 = \dfrac{P}{A_{min}}$

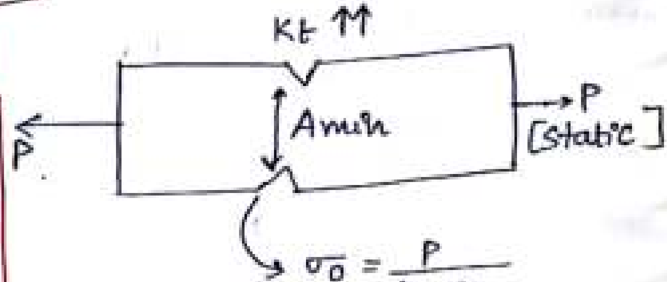
Increase in Actual stress over nominal stress

$= K_f \sigma_0 - \sigma_0$

**Static loading**

$\sigma_{static} = K_t \cdot \sigma_0$

$K_t \uparrow\uparrow$




$\sigma_0 = \dfrac{P}{A_{min}}$

Increase in static stress over nominal stress $= K_t \sigma_0 - \sigma_0$

$$q = \frac{\text{Increase in actual stress over nominal stress}}{\text{Increase in static stress over nominal stress}}$$

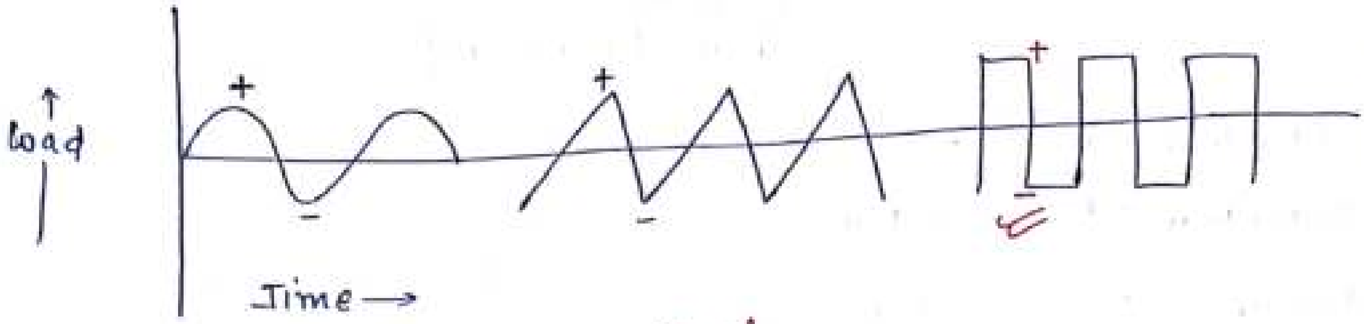
$$q = \frac{K_f \sigma_0 - \sigma_0}{K_t \sigma_0 - \sigma_0}$$

$$q = \frac{K_f - 1}{K_t - 1}$$

$$0 \le q \le 1$$

if $q = 0 \Rightarrow K_f = 1 \rightarrow$ Notch is not sensitive

$q = 1 \Rightarrow K_f = K_t \rightarrow$ Notch is fully sensitive.




Bending/Axial

$\sigma_e = k_a k_b k_c \text{ - - - - - - - - - } \sigma_e^*$

$\tau_e = k_a \cdot k_b \cdot k_c \text{ . - - - - - - - - - - } \sigma_e^*$ ~~$\tau_e^*$~~

Twisting

NEW CHAPTER

Gears [spur Gear]

Addendum 'a' = 1m

Dedendum 'd' = 1.157 m

clearance 'C' = .157m

Face width 'b' = 10 m

The aim of this Topic is to determine Module of gear.

$\boxed{P_c = \pi m}$

$P_d = \frac{1}{m}$

$P_c \cdot P_d = \pi$

$m = \frac{D}{z}$ ← no. of Teeth

T = Torque

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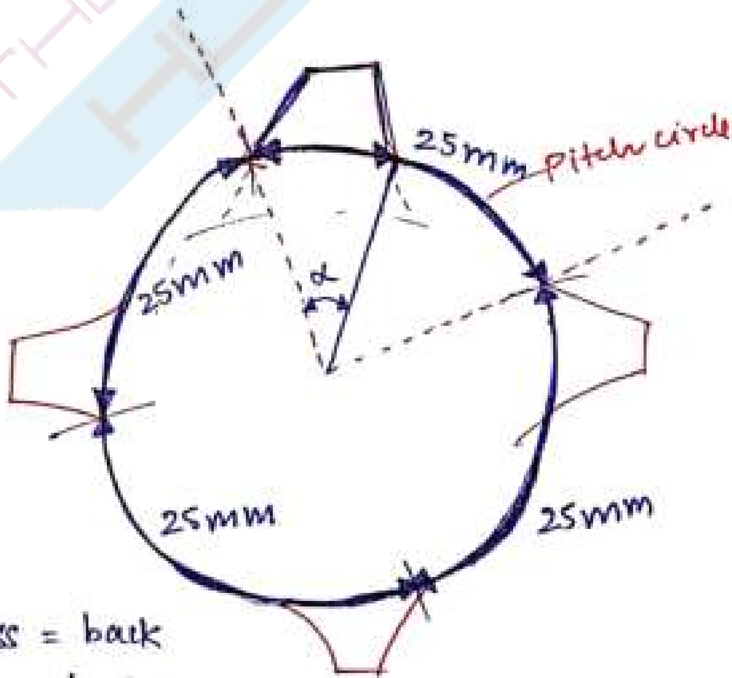
CIRCULAR PITCH

$P_c = \pi m = \frac{\pi D}{z}$

eg :- $\pi D = 100$ mm

$z = 4$

$P_c = 25$ mm




Tooth thickness + Tooth space = $P_c$

Tooth space - Tooth thickness = back lash

Tooth space ≈ Tooth thickness

$$\boxed{\text{Tooth thickness} = \text{Tooth space} = \frac{P_c}{2}}$$

when two gears are meshing together, their circular pitch must be equal.

$$\boxed{\alpha = \frac{360}{2 \times Z}}$$

Angle cover by Thickness / space on centre.

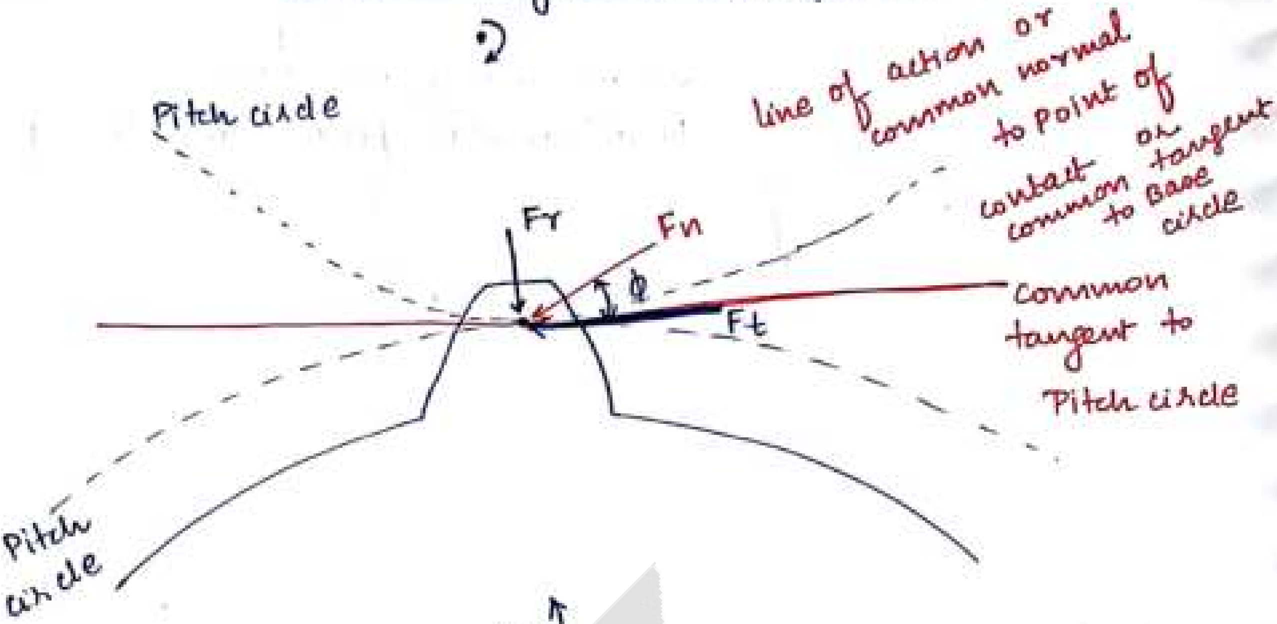
$P_{c_1} = P_{c_2}$

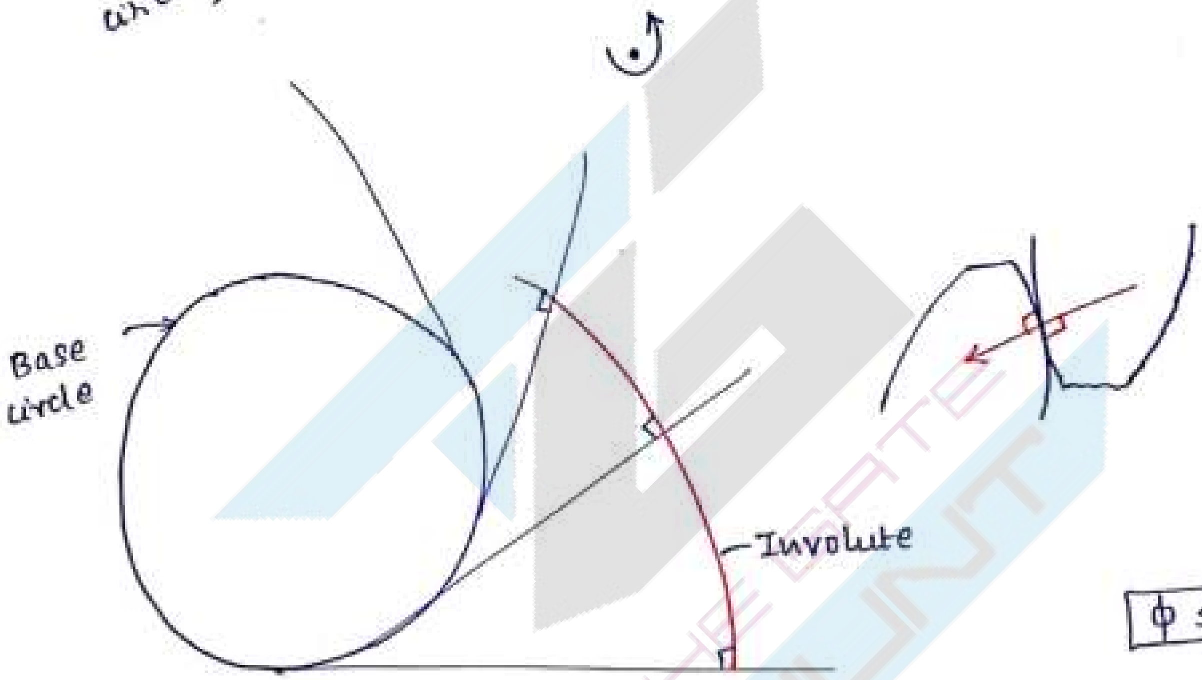
$\pi m_1 = \pi m_2$

$\boxed{m_1 = m_2}$

15/12/2016

## Force analysis used for Gears :-




$\boxed{\phi \le 25^\circ}$

$\phi$ = Pressure angle

$F_t = F_n \cos\phi$

$F_n = \frac{F_t}{\cos\phi}$ , $F_r = F_n \sin\phi$

$F_r = F_t \tan\phi$

$T = F_t \times R$

$$\boxed{F_t = \frac{2T}{D}}$$

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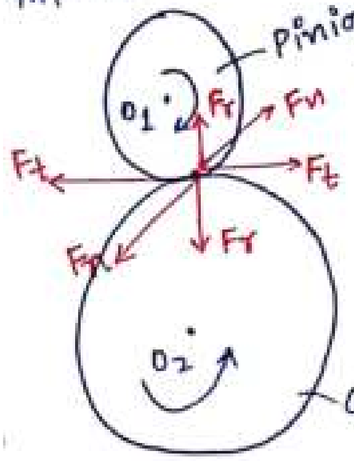
$\phi$
$m_1 = m_2$




Pinion ↳ [Prime mover]
$P_1, N_1$

∵ Inertia ↓
starting Torque loss
for Power
transmission

Pinion

$$P_1 = \frac{2\pi N_1 T_1}{60}$$

$T_1$ = known

$$F_t = \frac{2T_1}{D_1}, \quad D_1 = mZ_1$$

$m, Z_1$

$$F_t = \frac{2T_1}{mZ_1}$$

$\boxed{F_t = \text{known}}$

$$F_n = \frac{F_t}{\cos\phi}, \quad F_r = F_t \tan\phi$$

$F_n, F_r$ are known

Gear

$$T_2 = F_t \times \frac{D_2}{2}$$

$$T_2 = F_t \times \frac{mZ_2}{2}$$

$T_2$ = known

Law of gearing

$$\boxed{G.R. = G \geq 1 = \frac{D_2}{D_1} = \frac{Z_2}{Z_1} = \frac{N_1}{N_2} = \text{Fixed}}$$

case(1) if $\eta_m = 100\%$

$$P_2 = P_1$$

$$\frac{2\pi N_2 T_2}{60} = \frac{2\pi N_1 T_1}{60}$$

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{only when } \eta_m = 100\%$$

$$\boxed{G = \frac{D_2}{D_1} = \frac{Z_2}{Z_1} = \frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

when $\eta_m = 100\%$

Case II
when $\eta_m \neq 100\%$

$P_2 = \eta_m \cdot P_1$

Real Torque

$$\frac{2\pi T_2 N_2}{60} = \eta_m \frac{2\pi N_1 T_1}{60}$$

$$\frac{N_1}{N_2} = \frac{T_2}{\eta_m \cdot T_1}$$

$$\boxed{G.R. = \frac{D_2}{D_1} = \frac{Z_2}{Z_1} = \frac{N_1}{N_2} = \frac{T_2}{\eta_m T_1}}$$

$T_2 \neq F_t \cdot \frac{D_2}{2}$

$\downarrow P = T \downarrow \cdot \omega$

Torque loss $= \frac{F_t \cdot D_2}{2} - T_2$

Resultant ~~Torq~~ force on Pinion $= F_n$

" " " Gear $= F_n$

WORK

Gate book

Design - Gear

5.3 → $1\,kW = P$ $N = 1440$ rpm

$T = 56.36$ NM

$10:1$ $\frac{N_2}{N_1}$

$T_1$ $T_2$

$\frac{2\pi NT}{60} = P$

$\eta_m^2$ $N_1 N_2$

$$\frac{N_1}{N_2} = \frac{T_2}{\eta_m \cdot T_1}$$

$$10 : 1 = \frac{56.36}{\eta_m T_1}$$

$P_{in} = \frac{2\pi N_1 T_1}{60}$

$T_1 = 6.63$

$\eta_m =$

5.19 $T_1 = 20$ $N_1 = 30$ rev/s $\phi = 20°$

$T_2 = 40$ $P = 20 \times 10^3 W$ Full-depth system

$m = 5mm$

17 21
18
19
20
21

SIR $l = 19 mm$

Power $= 2\pi n_1 T_1 = \frac{2\pi N_1 T_1}{60} = T_1 \omega_1$

$20 \times 10^3 = 2\pi (30) T_1$

$T_1 = 106.1 N\text{-}m$

$F_t = \frac{2 T_1}{D_1} = \frac{2 T_1}{m z_1} = \frac{2 \times 106.1}{5 \times 20 \times 10^{-3}}$

$F_t = 2122 N$

$F_n = \frac{F_t}{\cos\phi} = 2258 N$

5.23 $20°$ $m = 4mm$ $P = 15 kW$ $b = 25mm$

21 teeth $N = 960 rpm$

SIR $P = \frac{2\pi N T}{60}$

$15 \times 10^3 = \frac{2\pi (960) T}{60}$

$T = 149.2 N$

$F_t = \frac{2T}{mz} = \frac{2 \times 149.2}{4 \times 21 \times 10^{-3}} = 3552 N$

5.29 $D = 50mm$ $N = 200$ rad/s $P = 3kW$ $\phi = 20°$

$F_n$

$\frac{2\pi N T}{60} = P$

$F_t = \frac{2T}{D} = \frac{2 \times T}{D}$

$F_r = F_t \times \tan\phi$

$F_r = F_n \sin\phi$

?

$P = T\omega$

$3\times10^3 = T\times200$

$T = 15\ N\text{-}m$

$F_t = \frac{2T}{D} = \frac{2\times15}{.05} = 600N$

$F_n = \frac{F_t}{\cos\phi} = 638N$

* centre distance वाले → S.12, S.13, S.14, S.16, S.17, S.18 → Do it.

S.22

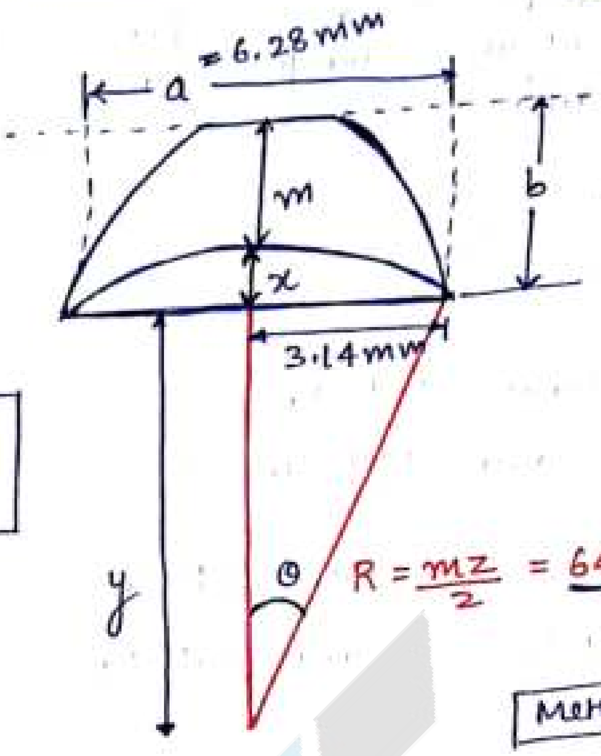
$m = 4$

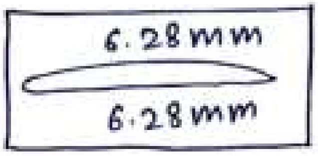

N.P.

5.22

$a \approx \frac{P_c}{2}$

$a = 6.28mm$




$b = m + x$

$x = R - y$

$y = \sqrt{64^2 - 3.14^2}$

$y = 63.92 mm$

$x = .08$

$b = 4.08$

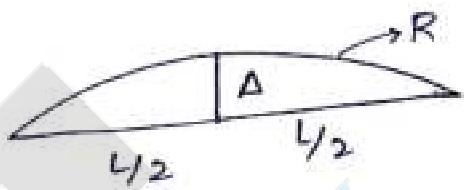
Method-2

$\theta = \frac{360}{2 \times 2 \times 32} = \frac{\alpha}{2}$

$\theta = 2.8°$

$y = R\cos\theta$

Method-3




$\left(\frac{L}{2}\right)^2 = \Delta(2R - \Delta)$.

$(3.14)^2 = x(2 \times 64 - x)$

$x = .08 mm$

5.11 $\phi = 20°$

$T_{min} = \cancel{\frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$

$G = ?$

Conclusion → ① More the addendum, more the chance of interference.

② hence there is max$^m$. chance of interference in case of Rack and pinion arrangement. so always design any gear by assuming Rack & pinion to avoid interference.

$$T_{min}\ \text{Rack or Pinion} = \frac{2a_r}{\sin^2\phi} = \frac{2\times1}{\sin^2 20^\circ} = 17.09^\circ \approx 18 \text{ Teeth}$$

$a_r = 1$ Full depth

$a_s = \cdot 8$ Stub depth

Full depth, $\phi = 20^\circ$ $T_{min}$ = 18 teeth

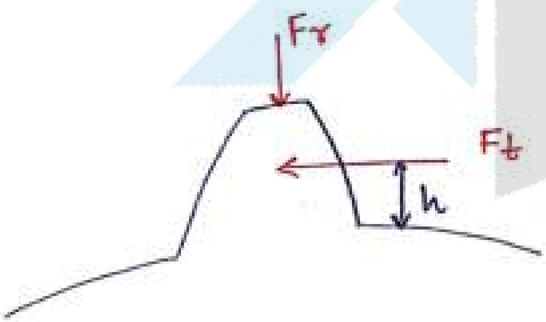
Best

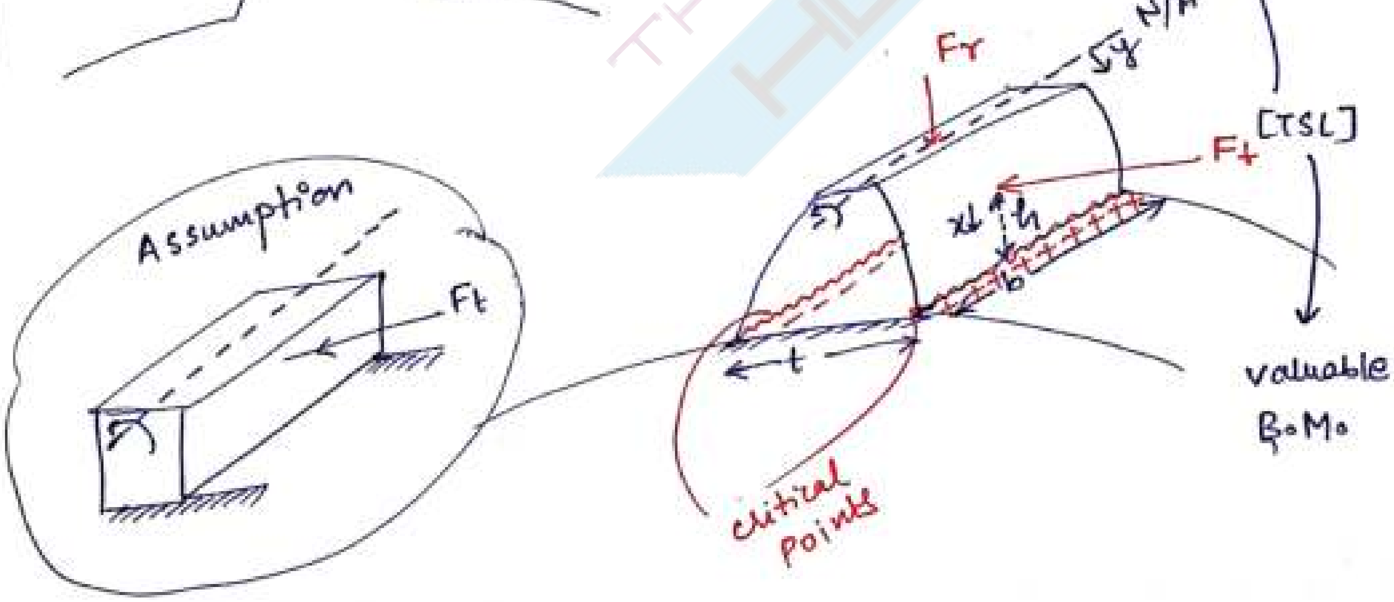
Stub tooth, $\phi = 20^\circ$ $T_{min}$ = 14 teeth

Full depth, $\phi = 14\tfrac{1}{2}^\circ$. $T_{min}$ = 32 teeth

stub tooth, $\phi = 14\tfrac{1}{2}^\circ$ $T_{min}$ = 26 teeth.

## DESIGN OF SPUR GEAR



Conclusion :- ① Due to axial compressive force $F_r$, gear tooth is subjected to compressive stresses.


② Due to shear force $F_t$, gear tooth is subjected to direct shear stresses. (TSL effect).

③ Due to variable Bending stress ( $F_t \times x$) gear tooth is subjected to Bending stresses.

④ For the safe design of the gear tooth, the effect of direct shear and compressive stresses are neglected, only Bending stresses will be taken into consideration.

$$(\sigma_b)_{max} = \frac{M_{max} \cdot y_{max}}{I_{NA}}$$

$$M_{max} = F_t \cdot h$$

$$I_{NA} = \frac{bt^3}{12}$$

$$y_{max} = \frac{t}{2}$$

$$(\sigma_b)_{max} = \frac{6F_t \cdot h}{bt^2}$$

Safe condn.

$$(\sigma_b)_{max} \leq \sigma_{per}.$$

$$\frac{6F_t h}{bt^2} \leq \sigma_{per}.$$

$$(F_t)_{max} = \frac{bt^2 m}{6h\,m} \cdot \sigma_{per}.$$

$$\frac{t^2}{6hm} = Y \text{ lewis form factor}$$

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$$\boxed{(F_t)_{max} = b \cdot m \cdot Y [\sigma_b]_{per}} \Rightarrow \text{Lewis equation.}$$

$\Downarrow$

Beam strength of gear tooth

Safe cond.

$$\boxed{F_{act} \le (F_t)_{max}}$$

* **BEAM STRENGTH** :- It is defined as the max^m. value of the tangential load that a gear tooth can bear without any Bending.

→ Lewis Form factor
or
Form factor
or
Tooth Geometry factor :-

$$Y = \pi y$$

$y$ = Tooth form factor

$$y = \left(.154 - \frac{.912}{z}\right) \text{ for full depth, } \phi = 20^\circ.$$

$$Y = \pi\left(.154 - \frac{.912}{z}\right) \text{ for full depth, } \phi = 20^\circ$$

$Y$ (L.F.F.) depends upon no. of teeth, Geometry of the tooth profile, pressure angle.

$$z_p < z_G$$

$$Y_p < Y_G.$$

$(F_t)_{max} = b m Y [\sigma_b]_{perm}$

Conclusion :-

① → weaker Gear is a Gear which has minimum value of the Beam strength and always design for weaker gear

② → when pinion and gear, both are made of same material then $[\sigma_b]_P = [\sigma_b]_G$

$$Y_P < Y_G$$

$$(F_t)_{max_P} < (F_t)_{max_G}$$

hence, pinion is weaker so design for pinion in this case.

③ → when pinion and gear both are made of different material then design for the gear which has minm. value of the product.

$$Y [\sigma_b]_{per}$$

Actual load :-

$$\text{Power} = \frac{2\pi N T}{60}$$

$$T = \text{Known}$$

$$F_t = \frac{2T}{D}$$

$$F_t = \frac{2T}{mZ}$$

[Static Load] ← $F_t$ = Known

Safe condn.

$$F_{act} \le (F_t)_{max}$$

$$F_{act} = F_{dynamic}$$

$$F_{dynamic} = F_{static} \times C_v \times \delta$$

$F_{dynamic} > F_{static}$

$$F_d = F_t \times C_v \times \delta$$

$C_v$ = velocity factor

$\delta$ = service factor

$$F_d = F_t \times C_v \times \delta$$

$$F_t C_v \delta \le (F_t)_{max}$$

***

$$\boxed{(F_t)\ C_v\ \delta \le b m Y [\sigma_b]_{per}}$$

$C_v = \frac{3+v}{3}$ when $v \le 10$ m/s

$C_v = \frac{6+v}{6}$ when $v > 10$ m/s

Gate book

(20) $m = 3$

$T_1 = 16$

$b = 36$ mm

$\phi = 20°$

3kW $N_2 =$

$$F_t \cdot C_v \cdot \delta \le b m Y [\sigma_b]_{per}$$

(95%)

$$994.7 \times 1.5 \le 36 \times 3 \times .3 [\sigma_b]_{per}$$

$$\boxed{[\sigma_b]_{per} = 46 \text{MPa}}$$

Power = $2\pi N T$

$3 \times 10^3 = 2\pi (20) T$

$T = 23.87$ N-m

$$F_t = \frac{2T}{m z} = 994.7 \text{N}$$

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(24) Gate Book

$\phi = 20°$
$m = 4$
$z_1 = 21$
$P = 15$
$N = 9.0$

TOM — M
FM — EM
SOM — TOM 1
T/M — TOM 2
PRo. — PP

(17)

SIR $3552 \times 1.5 \le 25 \times 4 \times .32 [\sigma_b] = \underline{166.5 \text{ MPa}}$

$$F_t \cdot C_v \cdot \delta \le b\, m \cdot Y [\sigma_b]_{per}$$

$$994.7 \times 1.5 \le 36 \times 3 \times .3 [\sigma_b]_{per}$$

$$\boxed{(\sigma_b)_{per} = 46 \text{ MPa}}$$

## * Wear strength of Gear Tooth.

It is defined as the maxm. value of the load that a gear tooth can wear without any wear failure.

wear strength is always check for pinion only because pinion is subjected to more wear.

$$F_w = D_p \cdot b \cdot Q \cdot K$$

wear strength

$D_p$ = pitch circle dia. of the pinion

$b$ = face width

$Q$ = Ratio factor

$$Q = \frac{2G}{G \pm 1}$$

(+) — External gear.

(−) — Internal gear.

K = material combination factor

$$\boxed{K = \frac{\sigma_{es}^2 \sin\phi \left[\frac{1}{E_P} + \frac{1}{E_G}\right]}{1.4}}$$

$$\left[\begin{array}{l} K \propto (BHN)^2 \\ \\ K \propto (S_{ut})^2 \end{array}\right]$$

$\sigma_{es}$ = surface endurance limit of the gear tooth.

$E_P$ and $E_G$ = Young's Modulus of pinion and gear.

$\phi$ = Pressure angle.

Safe condition

$$F_{act} \le F_W$$

$$\boxed{F_t \cdot C_v \cdot \delta \le D_p \cdot b \cdot Q \cdot K}$$

Hence safe from wear

Practical case

$$F_W \geq (F_t)_{max} \geq F_{act}.$$

wear strength $\geq$ Beam strength $\geq$ Actual load.

eg:

$F_W$ = 15 kN

$(F_t)_{max}$ = 20 kN

Power = ?

$F_{act} \leq 15$

$F_t \cdot C_v \cdot 8 \leq 15$

$F_t$ = known

T = known

Power = known

## * Assumptions made in Lewis's Equation :-

① Gear Tooth assumed as a cantilever Beam fixed at the root portion.

② The effect of direct shear and compressive stresses are neglected.

③ Gear Tooth assumed as a prismatic throughout.

④ The effect of stress concentration factors are neglected.

⑤ Inertia of the Rotating part neglected.

⑥ Deflection of the Tooth under load neglected.

⑦ Errors in Tooth manufacturing and spacing are neglected.

⑧ Contact Ratio assumed as 1 (one).

all these assumptions are the reason for the dynamic loading.

# Type of Wear

*Abrasive wear → ① These type of wear occurs b/w meshing gear surface due to presence of foreign material by the dust deposit or something by the ~~[illegible]~~ lubricant.

② occurs more in open gears.

* Scoring and Scuffing ~~gear~~ Wear :- This type of wear occurs between meshing gear surface due to failure of lubrication.

Scoring → Scratches in sliding dirn

Scuffing → welding due to heating.

* Corrosive wear :→ Due to chemical Rxn between lubricant and mating/meshing surfaces. These type of wear occurs

* pitting :→ This type of wear occurs between meshing gear surface due to repeated stress occur under cyclic loading.

New chapter

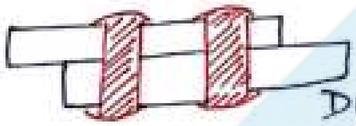
# Riveted Joint [Permanent Joint]

- Joint
  - Temporary
    - Cotter Joint
    - Keyed Joint
    - Bolted Joint
    - Screwed Joint
  - Permanent
    - Riveted Joint
      - Lap Joint
      - Butt Joint
        - Single strap B. J.
        - Double strap B. J.
    - Welded Joint

Strap

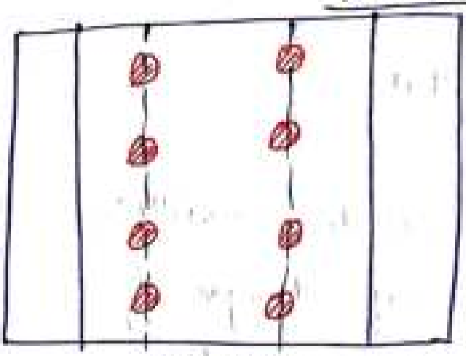
Butt Joint

F.V.



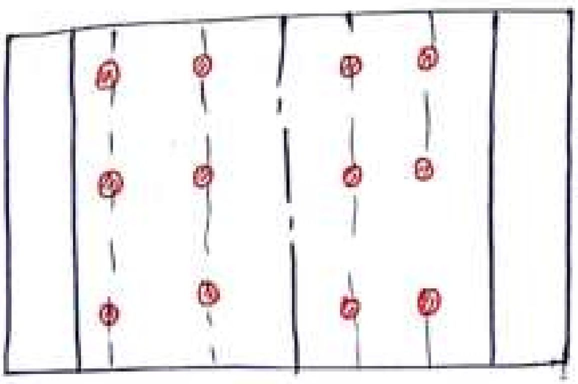
Double Row-lap Joint

B.R.L.J



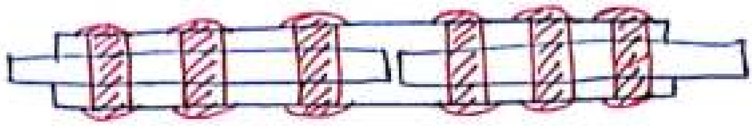
Top view

Double row - single strap B. J.



DRSSBJ

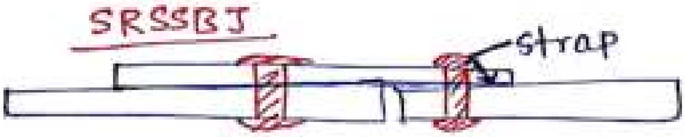
## Double strap B.J.



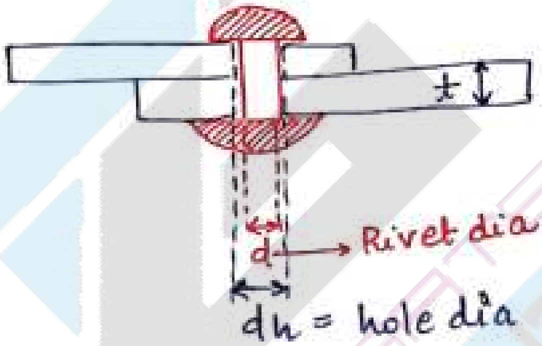
Tertiary Row - Double strap B.J.

TRDSBJ

SRSSBJ



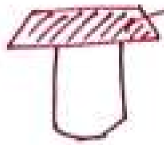

Head [Snap Head] ⇒ used in boilers
fatigue strength ↑


d = shank dia / Rivet dia.

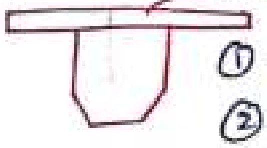
$$\boxed{\text{Riveted Joint} = \text{Rivet} + \text{Plate}}$$

For the safe design of the Rivet, shank dia D will be taken into consideration and for the safe design of the plate; hole dia. $D_h$ will be taken under consideration.



Pan head / cone head
① ship hull's / boiler
② Corrosive strength ↑

counter sunk head
[underwater working]



Flat head
① Thin Plates.
② structural work.

Unwin's formula ⇒ $\boxed{d = 6\sqrt{t}}$

m.m. m.m.

* Type of failure :-

✓ Lap Joint :-

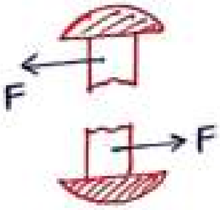
shearing [Single shear failure)]

only one Rivet

crushing

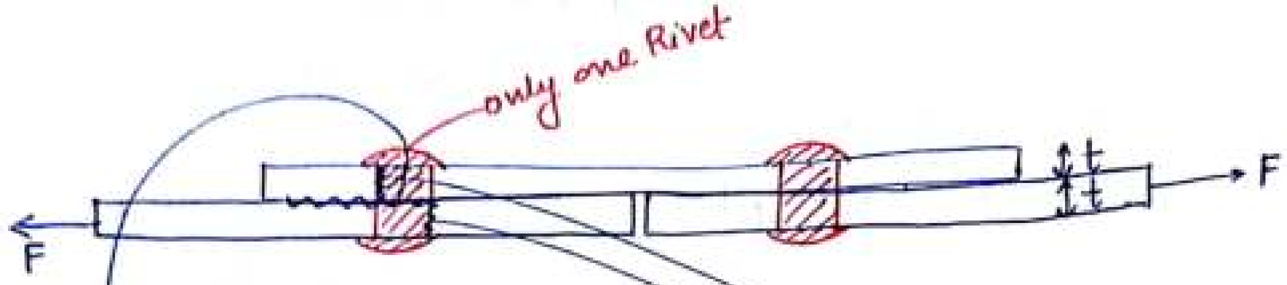


$$(\sigma_{ind})_{crush} = \frac{F}{dt}$$




$$\tau_{ind} = \frac{F}{\frac{\pi}{4}d^2}$$

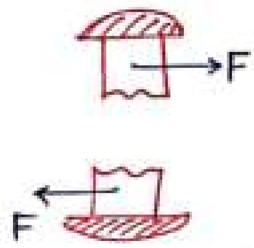
## Single strap B.J.




Shearing
[single shear failure]

Crushing "dt"

$$(\sigma_{ind})_{crush.} = \frac{F}{dt}$$

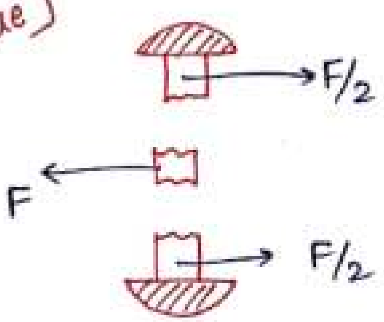



$$\tau_{ind} = \frac{F}{\frac{\pi}{4}d^2}$$

## Double strap B.J.


Shearing
[Double shear failure]

$$\tau_{ind} = \frac{F}{2 \cdot \frac{\pi}{4} d^2}$$




Crushing

$$\sigma_{ind\ crush\ max} = \frac{F}{dt}$$

$$\tau_{ind} = \frac{F}{k \cdot \frac{\pi}{4} d^2}$$

$K = 1 \rightarrow$ single shear

$K = 2 \rightarrow$ Double shear

$$(\tau_{ind})_{crush} = \frac{F}{dt}$$

For all riveted Joint

---

Case No. 1 finite Riveting

$$\tau_{ind} = \frac{P}{4 \cdot K \frac{\pi}{4} d^2}$$

safe condn.

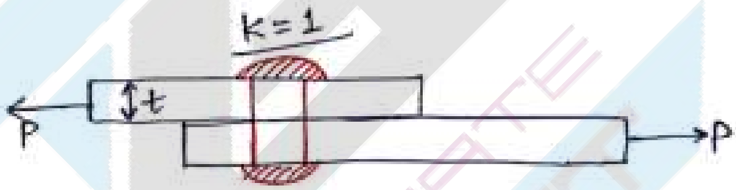
$$\tau_{ind} \le \tau_{pr}$$

$$\frac{P}{4 \cdot K \frac{\pi}{4} d^2} \le \tau_{pr}$$

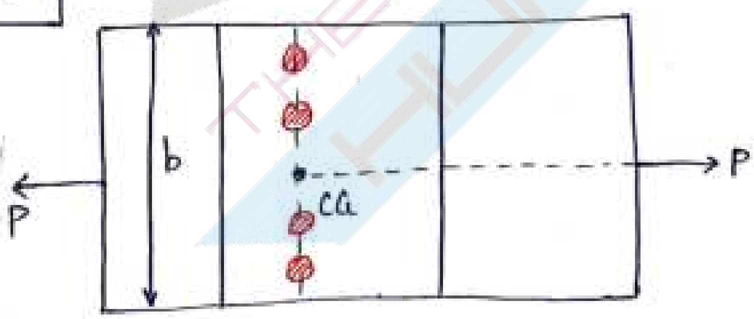
$$\boxed{P_{max} = 4 \cdot K \frac{\pi}{4} d^2 \tau_{pr}}$$

$\Downarrow$

shear strength of RJ/ Rivet




$F_{rivet} = P/4$.




Shear Design of Riveted Joint ~~oblique~~/ Rivet.

$$\sigma_{\substack{ind.\\crush}} = \frac{P}{4dt}$$

safe condn.

$$\sigma_{\substack{ind.\\crush}} \le \sigma_{pr}$$

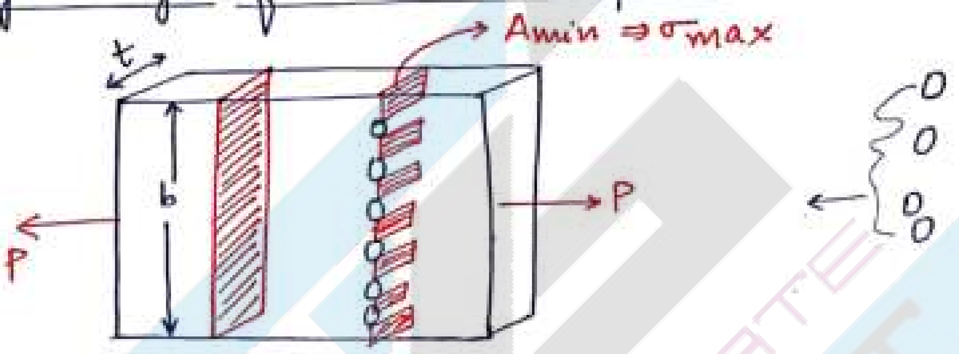
$$\frac{P}{4dt} \le \sigma_{pr}$$

$$\boxed{P_{max} = 4dt \cdot \sigma_{pr}}$$

⇓

crushing strength of R.J./Rivet.

## Tearing design of Riveted Joint/plate



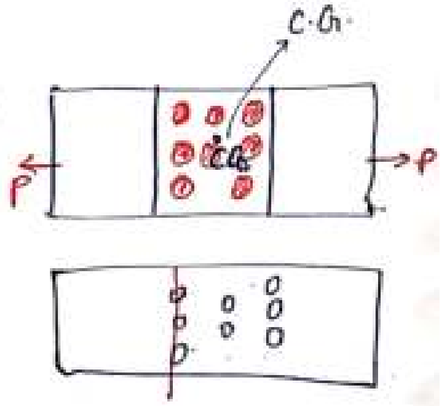

$$\sigma_{\substack{max\\Tearing}} = \frac{P}{(b-4d_h)t}$$

$$\boxed{P_{max} = (b-4d_h)t \cdot \sigma_{pr}}$$

⇓

Tearing strength of R.J./plate

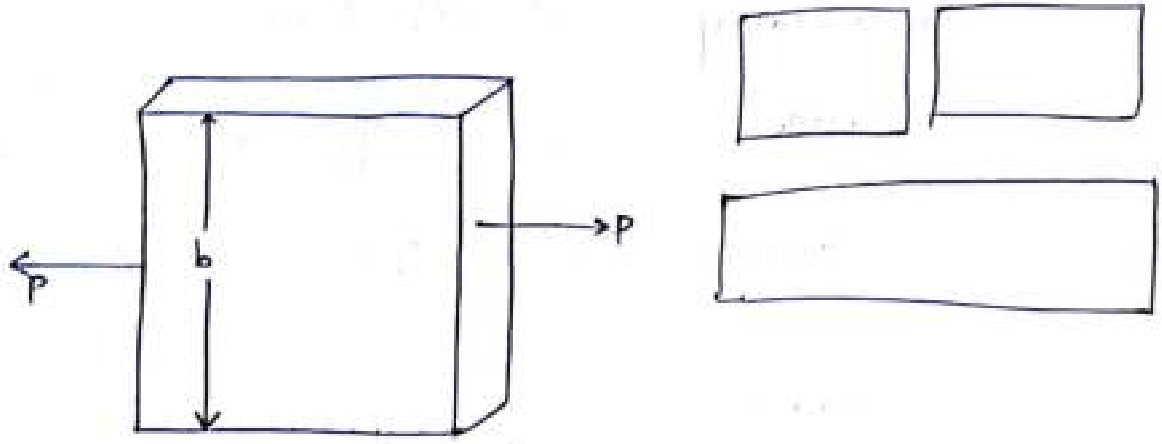
Ex:-




Actual strength of R.J. = min of [$(P_{max})_{shear}$, $(P_{max})_{crush}$, $(P_{max})_{Tearing}$]

—"— Rivet = min of [$(P_{max})_{shear}$, $(P_{max})_{crush}$].

## * Strength of the solid plate :-




$$\sigma_{ind} = \frac{P}{bt}$$

$$\boxed{(P_{max})_{solid} = bt \cdot \sigma_{pr}}$$

→ Best possible joint forever. for the comparison purpose only

$$\eta_{shearing} = \frac{(P_{max})_{shear}}{(P_{max})_{solid}} = \frac{4 \cdot \frac{\pi}{4} d^2 k \, \tau_{ind}}{bt \cdot \sigma_{pl}}$$

$$\eta_{crushing} = \frac{4 \cdot d \cdot \cancel{t} \, [\sigma_{pl}]_{Rivet}}{b\cancel{t} \, [\sigma_{pl}]_{plate}}$$

$$\eta_{Tearing} = \frac{(b - 4d_h)\cancel{t} \cdot \cancel{\sigma_{pr}}}{\cancel{bt} \cdot \cancel{\sigma_{pr}}} = 1 - \frac{4d_h}{b}$$

$$\eta_{actual} = \min. \text{ of } [\eta_{shearing}, \eta_{crushing}, \eta_{Tearing}]$$

Gate 3.12

$P_{max}$ crushing = $3 \cdot d \cdot t \cdot \sigma_{pr}$

$P_{max}$ Tearing = $(w - 3d_h) \cdot t \cdot \sigma_{pt}$

$P_{max}$ shear = $3 \cdot \frac{\pi}{4} d^2 \cdot \tau_{per}$

$P_{max}$ solid = $4\, l \cdot t \cdot \sigma_{per}$

3.14 $\sigma_{ys} = 200$ MPa

FOS = 2


Bolt will fail (shear) → single ∵ load sey paas hote hue.

M-10 ✓ 8.97 = D ✓

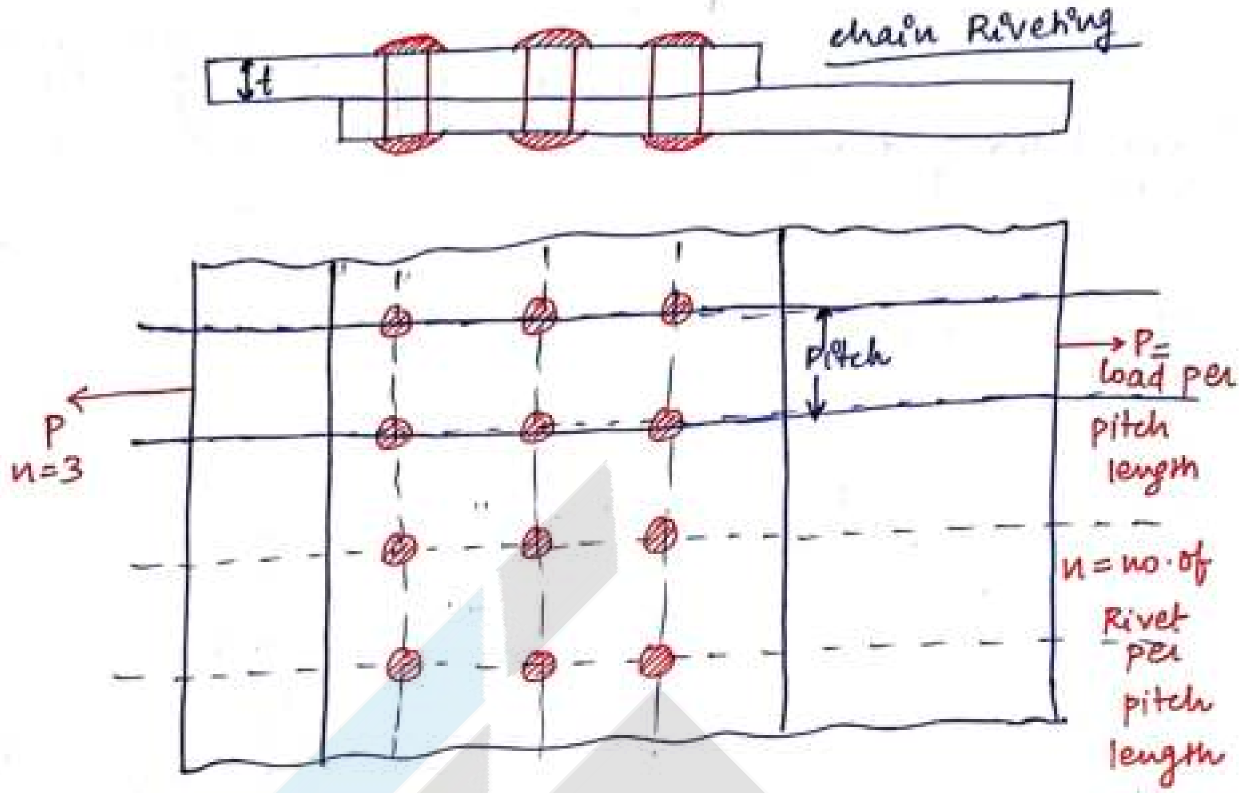
SIR

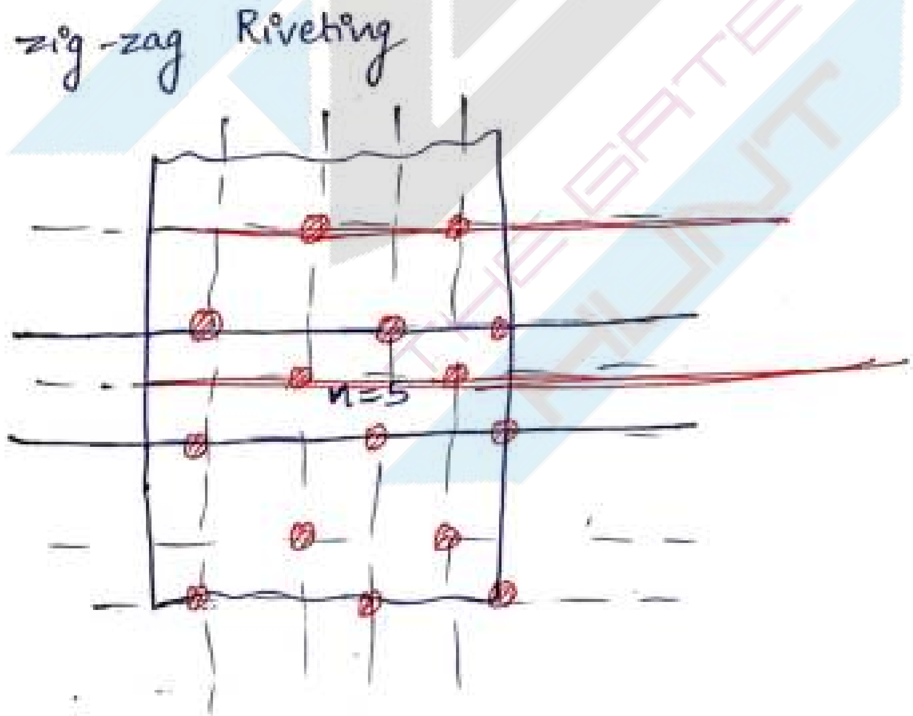
$$\frac{19/3}{\frac{\pi}{4} d^2} \le \frac{200}{2} \times 10^{-3}$$

$d \geq 8.9$ mm

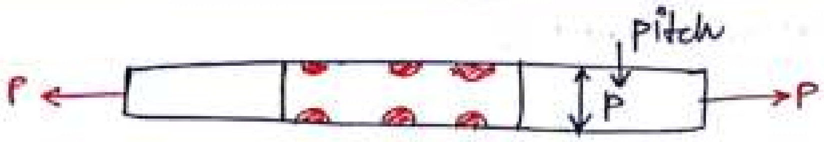
$\boxed{d = 10 \text{ mm}}$

## Case II → Case of Infinite Riveting



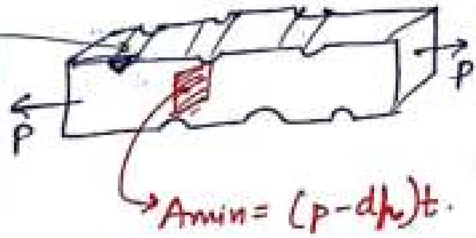




MOHIT KUMAR CHOUKSEY




$n = 3$

$F_{Rivet} = \frac{P}{n}$




$$P_{\max\ shear} = nk \cdot \frac{\pi}{4} d^2 \tau_{pl}$$

$$P_{\max\ crushing} = n \cdot d \cdot t \cdot \sigma_{pl}$$

$$P_{\max\ Tearing} = (p - d_h) t \cdot \sigma_{pr}$$

$$P_{\max\ solid} = p \cdot t \cdot \sigma_{pl}$$

ESE-2017

$$\boxed{\eta_{Tearing} = 1 - \frac{d_h}{p}}$$

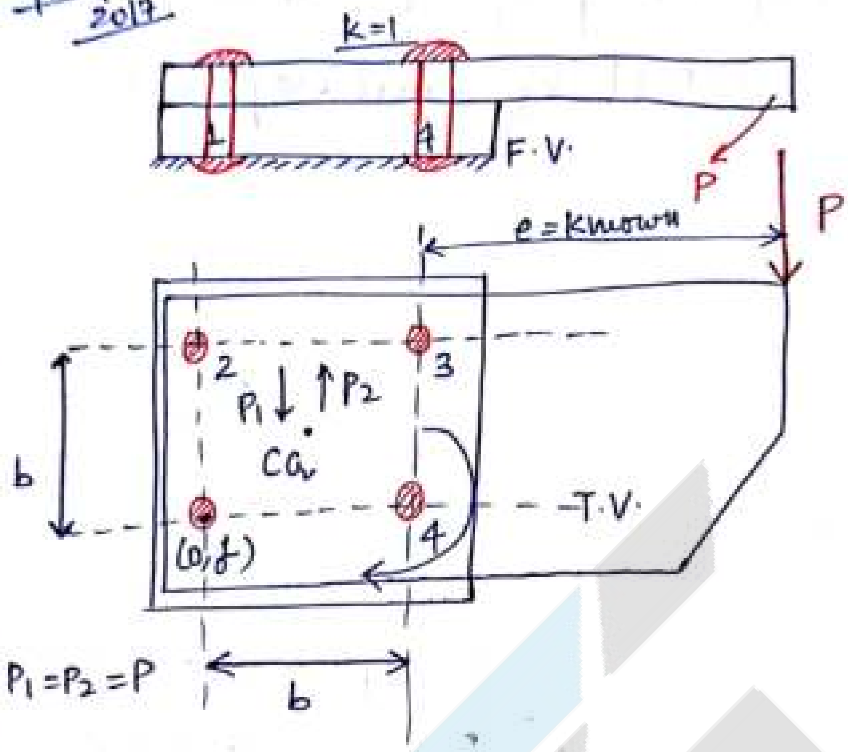
$$\eta_{shear} = \frac{nk\frac{\pi}{4}d^2 \tau_{pr}}{pt \cdot \sigma_{pr}}$$

$$\eta_{crushing} = \frac{n\,d\,t \cdot \sigma_{pl}}{pt \cdot \sigma_{pl}}$$

$$\eta_{tearing} = \frac{(p - d_h)\,t \cdot \sigma_{pl}}{pt \cdot \sigma_{pl}}$$

very very imp. Topic for gate 2017

# Design of Riveted / Bolted Joint :- under eccentric loading




$P_1 = P_2 = P$

**Solution →** **Step 1 →** Find out the C.G. of the group of Rivet / bolt.

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 + \cdots}{n} \Rightarrow \bar{X} = \frac{0+0+b+b}{4} = \frac{b}{2}$$

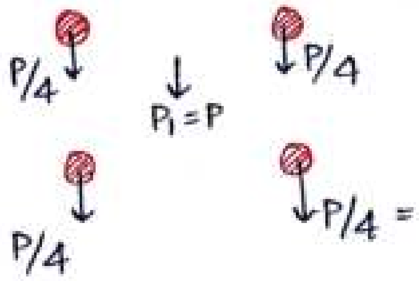
$$\bar{Y} = \frac{Y_1 + Y_2 + Y_3 + Y_4 + \cdots}{n} \Rightarrow \bar{Y} = \frac{0+b+b+0}{4} = \frac{b}{2}$$

**Step 2 →** Find out Eccentricity → is the distance b/w C.G. of the group of Rivet to the line of action of the load.

**Step 3 →** apply 2 equal & opposite forces passing through C.G. of the group of Rivet such that $P_1 = P_2 = P$

Step 4 → Effect of $P_1$

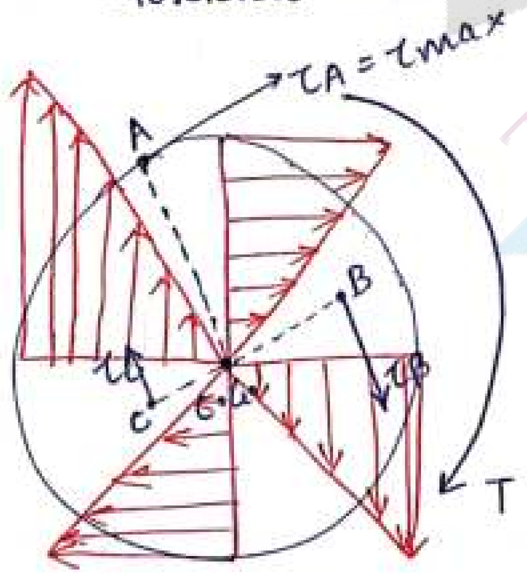
$P_1$ Passes through C.G. of the group of Rivet / Bolt. it result primary shear force induced of equal magnitude in each rivet / Bolt.

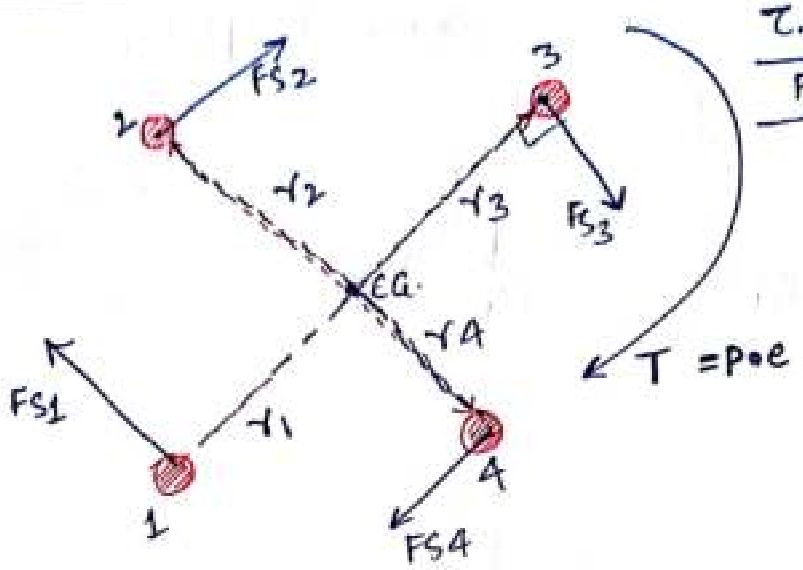



Step 5 →
$P_2$ and P causes a constant Twisting couple of magnitude $P \times e$ in clockwise direction about the C.G. of the group of rivet / Bolt which result twisting in the rivets.

Twisting in shaft

$\tau_{torsional} \propto r$






$\tau_{torsional} \propto r$

$F_S \propto r$

Secondaly sheal force $\longrightarrow$ $F_S$

$$F_{S_1} r_1 + F_{S_2} r_2 + F_{S_3} r_3 + F_{S_4} r_4 = P \cdot e$$

$F_S \propto r$

$$\frac{F_{S_1}}{r_1} = \frac{F_{S_2}}{r_2} = \frac{F_{S_3}}{r_3} = \frac{F_{S_4}}{r_4}$$

$$\frac{F_{S_1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right] = P \cdot e$$

$F_{S_1}$, $F_{S_2}$, $F_{S_3}$, $F_{S_4}$ are known

if $r_1 = r_2 = r_3 = r_4 = r$

$$F_{S_1} = F_{S_2} = F_{S_3} = F_{S_4} = \frac{P \cdot e}{4 \cdot r}$$

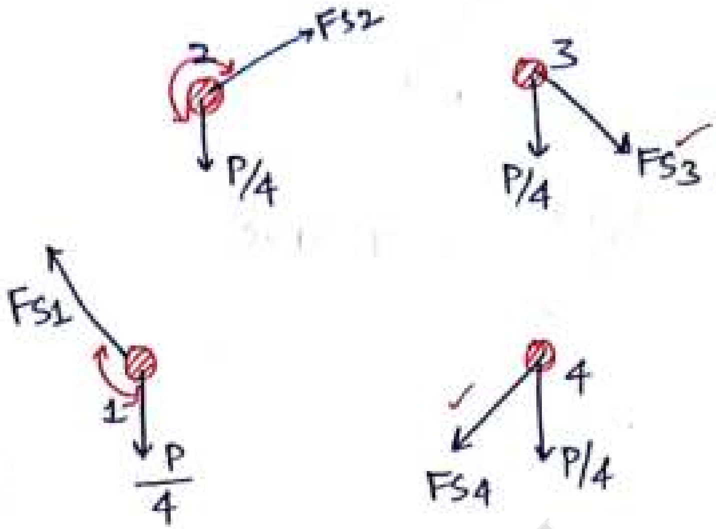
step 6 $\rightarrow$

N.P.

## Combined Effect

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\theta \downarrow \rightarrow R \uparrow$




## Critical Rivet or worst loaded Rivet


$$R_3 = \sqrt{\left(\frac{P}{4}\right)^2 + (F_{S_3})^2 + 2\left(\frac{P}{4}\right)(F_{S_3})\cos\theta}$$

maxm. Shear Force

$$\tau_{max} = \frac{R_3}{\frac{\pi}{4}d^2}$$

Safe condn

$$\frac{R_3}{\frac{\pi}{4}d^2} \leq \tau_{pr}$$

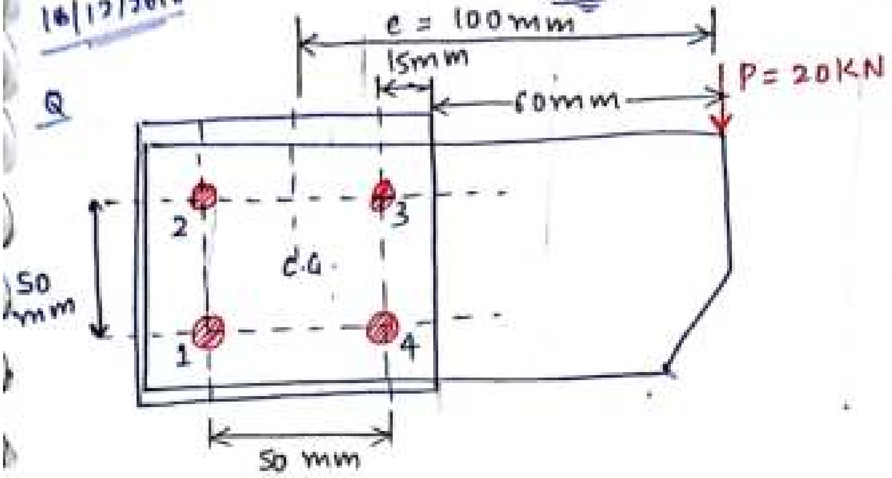
$$\boxed{d \geq \text{---}}$$

Answer

16/12/2016

"(माँ)"

**Q**




$\tau_{per} = 70\ MPa$

$d = ?$

**Effect of $P_1$**


**Combined effect**

Critical Rivet 3, 4


**Safe condn**

$$\frac{18.02}{\frac{\pi}{4}d^2} \le 70\times10^{-3}$$

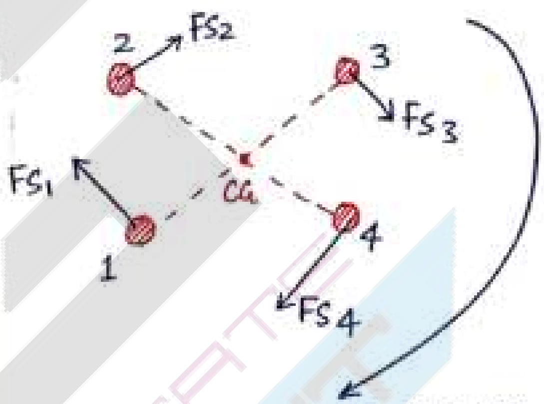
$$R_3 = \sqrt{5^2 + 14.14^2 + 2(5)(14.14)\cos 45}$$

$d \geqslant 18.1\ mm$

$d = 19\ mm$

$R_3 = 18.02\ KN$

**Effect of $P_2$ and P**

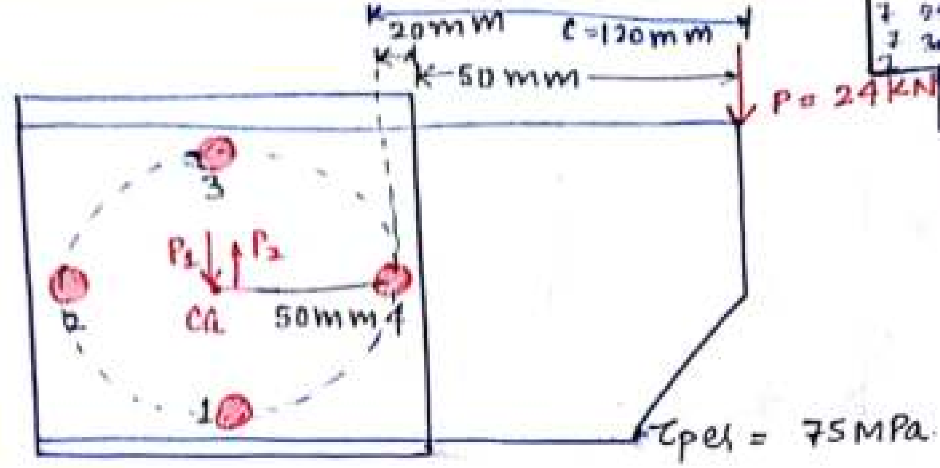



$P = 20\times100$ KN-mm

$r_1 = r_2 = r_3 = r_4 = 25\sqrt{2}$

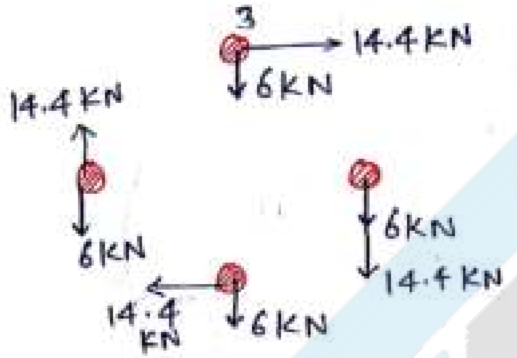
$$FS_1 = FS_2 = FS_3 = FS_4 = \frac{20\times100}{4\times25\sqrt{2}}$$

$= 14.14\ KN$

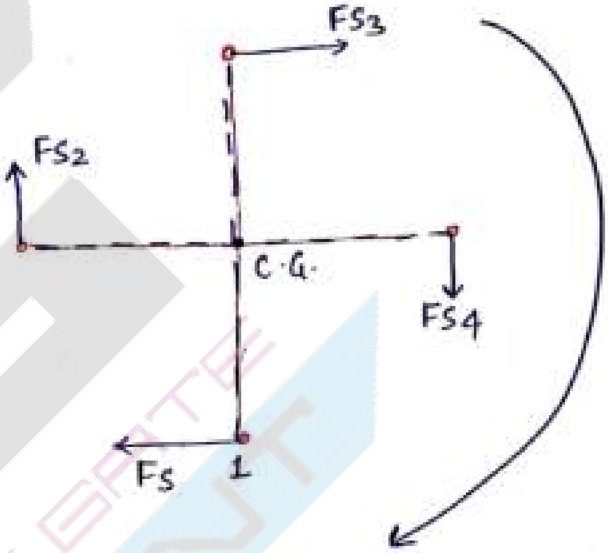



Sol:

**combined Effect of $P_1$**




**Effect of $P_2$ and P**




Critical Rivet ⇒ 4

$$T = 24 \times 120 \text{ kN-mm}$$

$$\frac{20.4}{\frac{\pi}{4}d^2} \leq 75 \times 10^{-3}$$

$R_4 = 20.4$ kN

$d \geq 18.6$

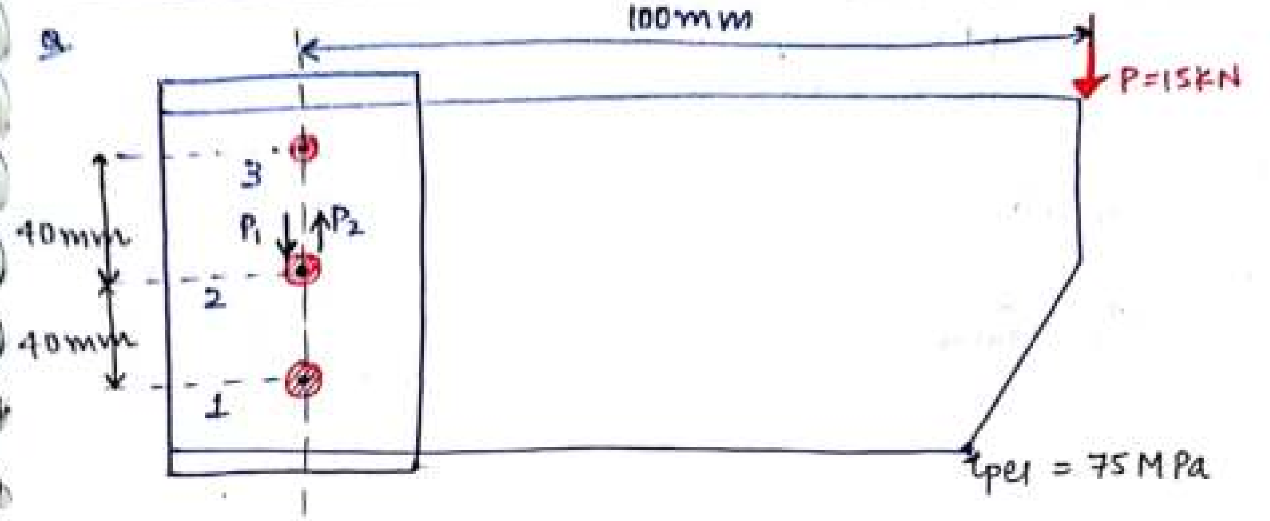
$\Rightarrow \boxed{d = 19 \text{ mm}}$

$r_1 = r_2 = r_3 = r_4 = 50$

$Fs_1 = Fs_2 = Fs_3 = Fs_4 =$

$$\frac{24 \times 120}{4 \times 50} = 14.4 \text{ kN}$$

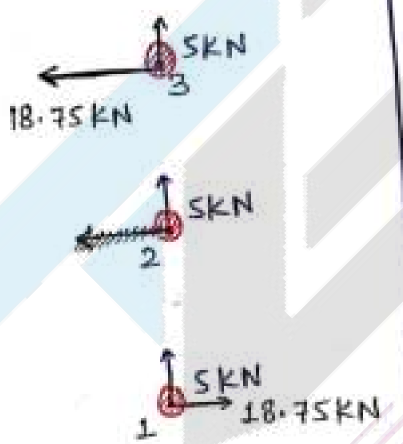
Q.



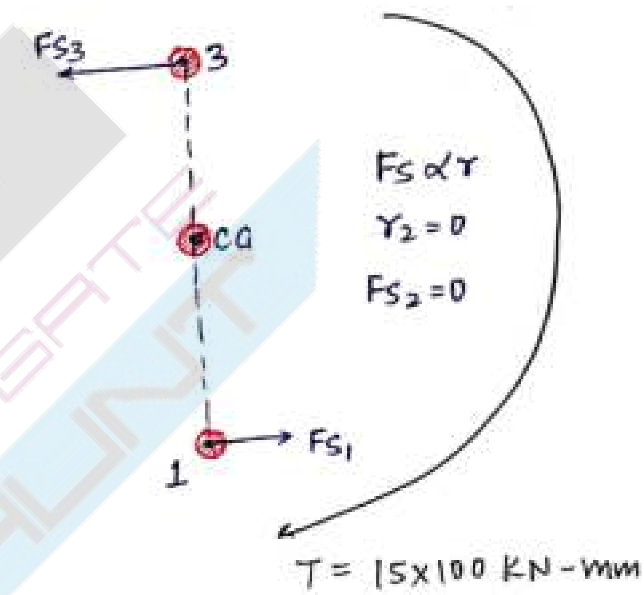

Find out the critical Rivet, Resultant force on each Rivet, safe diameter of the Rivet.

Sol

Effect of $P_1$




Effect of $P_2$ and P




critical Rivet 1,3

$$R_1 = R_3 = \sqrt{5^2 + 18.75^2}$$

$$R_1 = R_3 = 19.4 \text{ KN}$$

$$R_2 = 5 \text{ KN}$$

$$\frac{F_{S1}}{40}\left[40^2 + 0^2 + 40^2\right] = 15 \times 100$$

$$F_{S3} = F_{S1} = 18.75 \text{ KN}$$

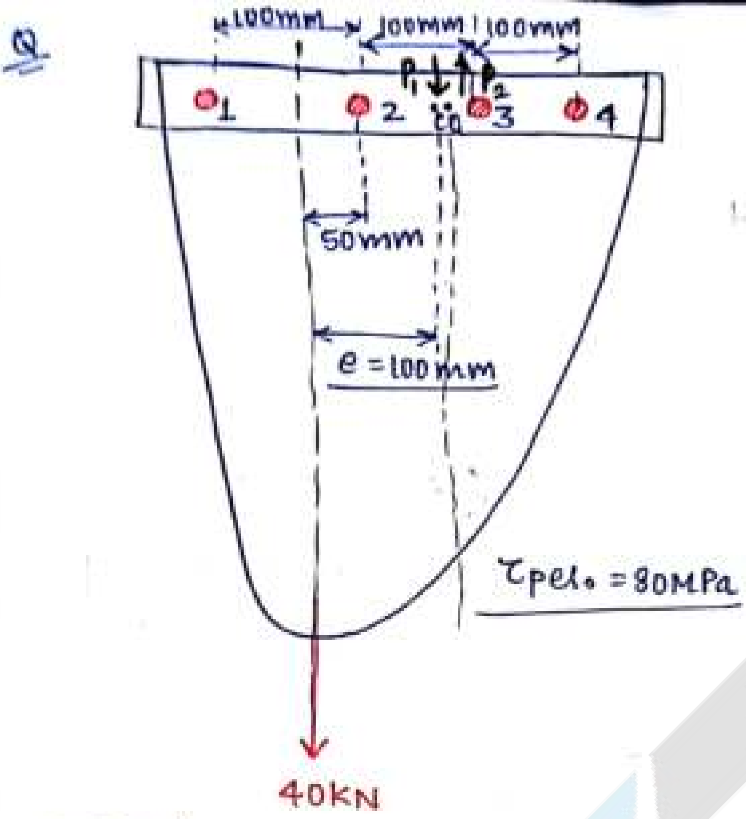
safe condn.

$$\frac{19.4}{\pi/4 d^2} \leq 75 \times 10^{-3}$$

$$d \geq 18.15$$

$$\boxed{d = 19 \text{ mm}}$$

Q



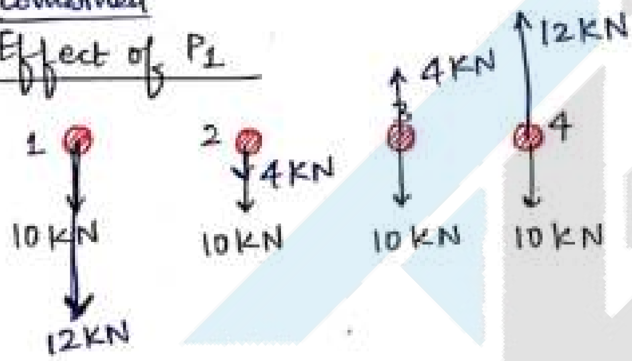

17 1 16 31
18 2 17 1
19 3 18 2
20 4 19 3
21 5 20 4 → Paper
22 6 21
23 7 22
24 8 23
25 9 24
26 10 25
27 11 26 19mm
28 12 27 4th is
29 13 28 critical
30 14 29
31 15 30

44 days

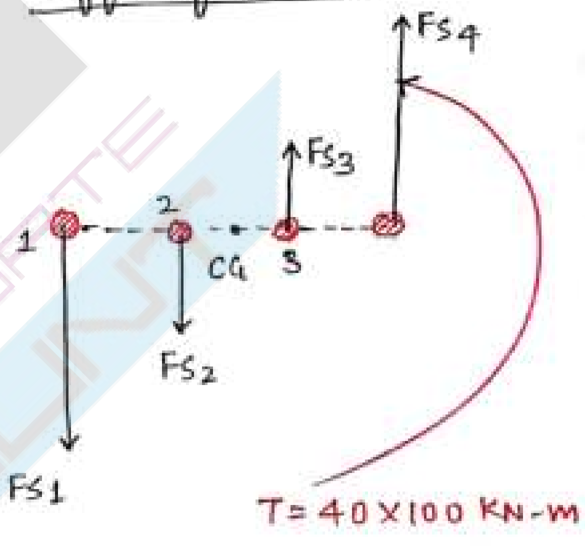
(13)

combined

Effect of $P_1$




Effect of $P_2$ and P




$R_1 = 22\,KN \rightarrow$ critical Rivet

$R_2 = 14\,KN$

$R_3 = 6\,KN$

$R_4 = 2\,KN$

$$\frac{F_{S1}}{150}\left[150^2 + 50^2 + 50^2 + 150^2\right] = 40 \times 100$$

$$\frac{22}{\frac{\pi}{4} d^2} \leq 80 \times 10^{-3}$$

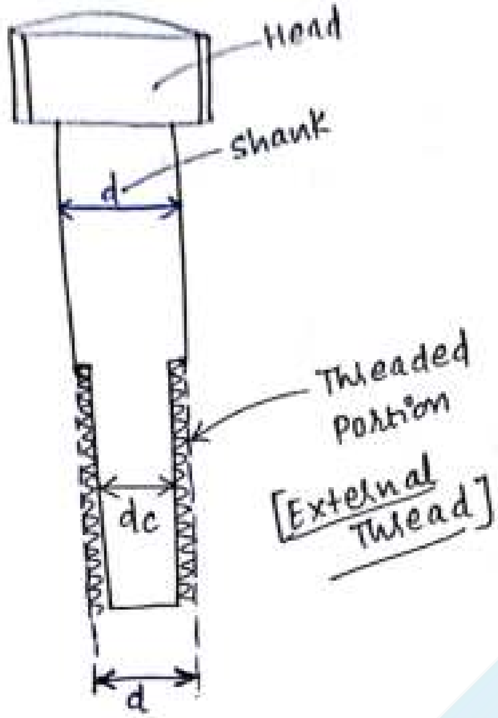
$F_{S1} = F_{S4} = 12\,KN$

$F_{S2} = F_{S3} = 4\,KN$

$d \geq 18.7$

$\boxed{d = 19mm}$

# NEW CHAPTER
## Bolted Joints




d = Major dia / shank dia / Nominal dia / dia

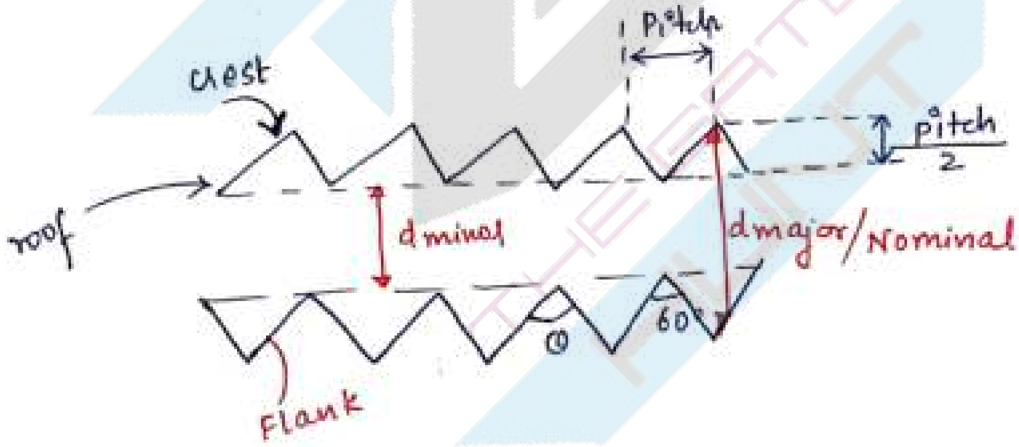
$d_c$ = core dia / Minor dia

$$\boxed{d_c = .84d}$$

Thread

→ External Thread

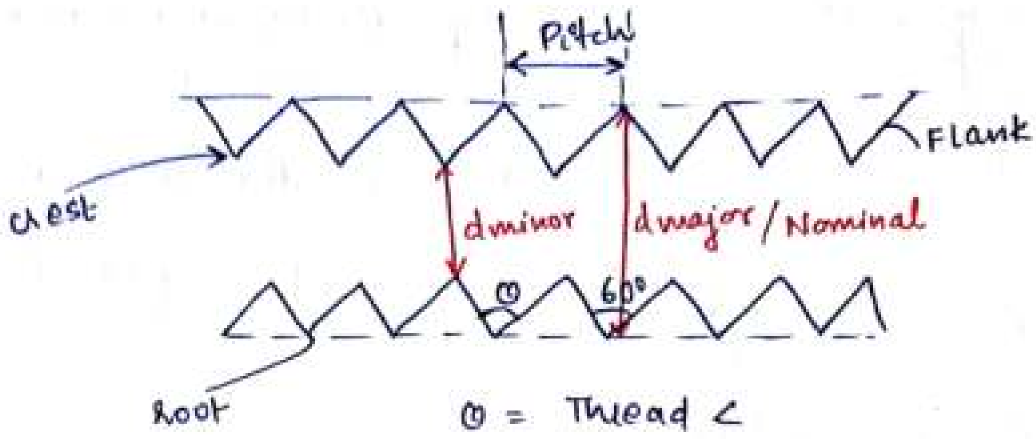
→ Internal Thread




θ = Thread angle

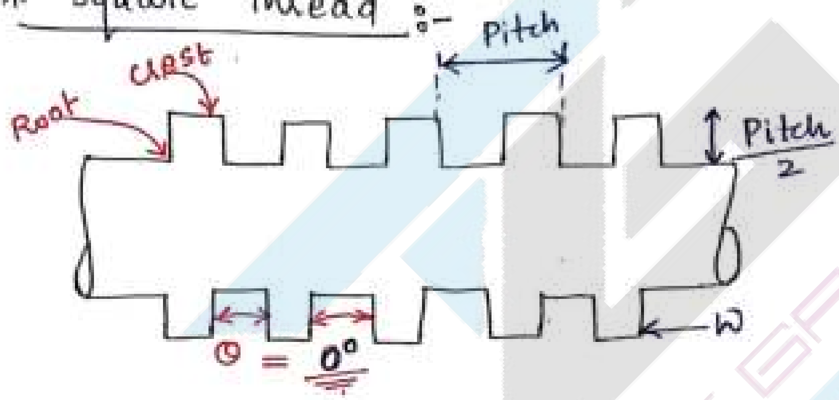
vee - shaped Thread

[used for fastening Purpose]




Vee - shape

* Square Thread :-




Square Thread

used in screw Jack

① $\eta_{max}$

(i) Mfg different

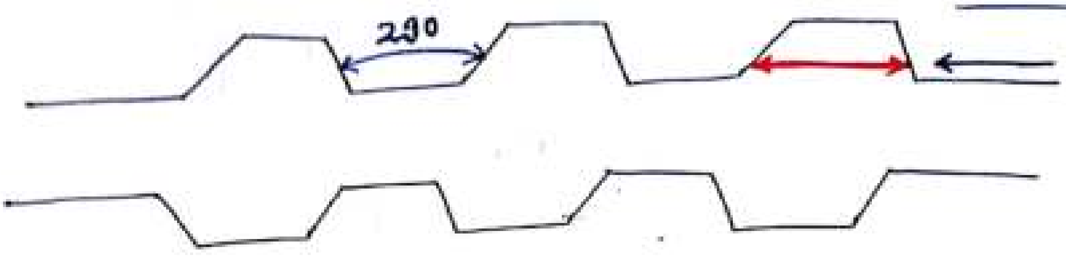
(ii) single Point tool

(iii) Cost ↑

(iv) strength ↓

→ strong as compared to square thread.

θ = 30° → Trapezoidal (पहले के Jamane में)

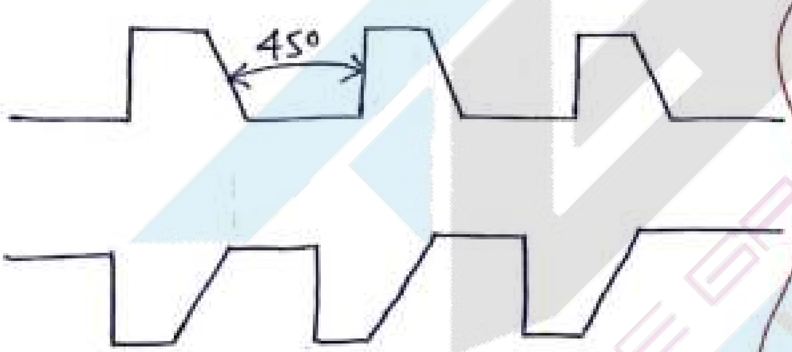



## ACME Thread

* ① Best thread to transmit Power in both direction.
* ② Most commonly used
* ③ Manufacturing simple.
* ④ cost ↓

(Lead screw of lathe)

worm and wheel → Reversible




① R.R ↓

② No. of start ↑

③ $\phi < \alpha$ → self lock

## Buttress thread

* Used to transmit power only in one direction.

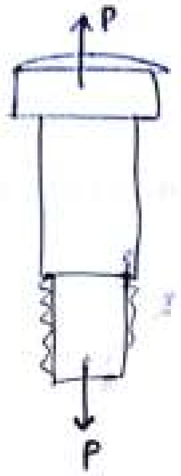
screw Jack / Power Press

$$\mu = \tan\phi$$

$$\boxed{\phi = \tan^{-1}(\mu)}$$

Bolt → shear

Aim → "Tension"




Find out safe diameter of

Bolt ⇒ Nominal dia $\boxed{d = ?}$

$$\sigma_{shank} = \frac{P}{\frac{\pi}{4}d^2}$$

$$\sigma_{max} = \sigma_{core} = \frac{P}{\frac{\pi}{4}d_c^2}$$

Safe condition

$$\sigma_{max} \le \sigma_{per}$$

$$\frac{P}{\frac{\pi}{4}d_c^2} \le \sigma_{per}$$

$d_c$ = known

$$d = \frac{d_c}{.84}$$ → This Result is of Databook hence late wale. $[10\text{mm} \le d \le 25\text{mm}]$

$\boxed{d = \text{known}}$

$d_c = .78d$ — $d > 25\text{mm}$

Conclusion :- (a) for the safe Design of the Bolt, core diameter '$d_c$' will be taken into consideration becoz core is the weakest portion of the Bolt.

(b) GATE

Bolt ≈ Rivet

No ~~de~~ Ka funda.

WK Gate book

3.18

$$\frac{19/3}{\frac{\pi}{4} d_c^2} \le 100$$

$d_c = 8.9$

$d = \frac{8.9}{.84} = 11. - -$

M10 - - M12

3.7 (a) d में कला ✓

$M_{20 \times 2}$ → 2mm Pitch [Fine Pitch]

↓

20mm ⟶ Bolt dia / Nominal dia / Magnitude

$M_{20}$ → coarse Pitch

10mm ✓

↓

20mm = Major dia

| Pitch → | 2mm | 4mm | 6mm | 8mm | 10mm |
|---|---|---|---|---|---|
| 20 mm | ✓ Fine Pitch | ✓ | ✓ | ✓ | ✓ $\boxed{\text{Coarse Pitch}}$ |
| 24mm | ✓ | × | ✓ | ✓ | × |

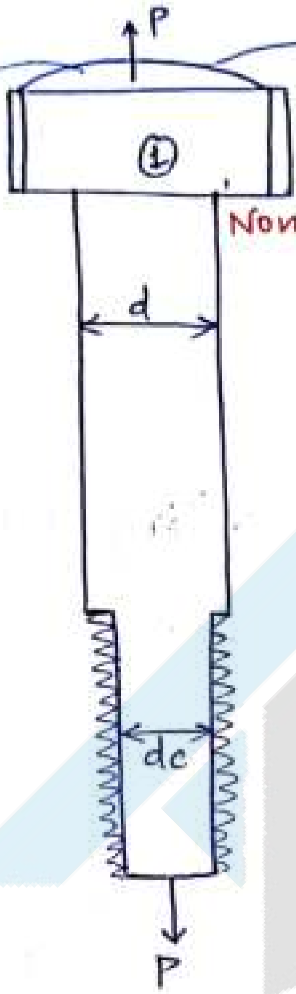
Fine Pitch

coarse pitch

## Uniform strength Bolt :-

A Bolt is said to be uniform strength Bolt when stress induced is equal in each point in the Bolt.

Method-2




Non uniform strength Bolt

$$\sigma_{shank} = \frac{P}{\frac{\pi}{4}d^2}$$

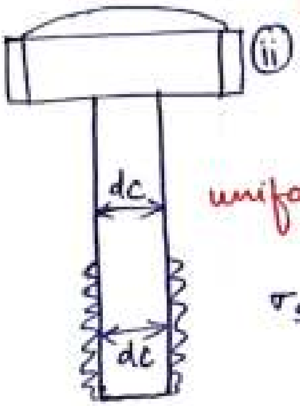
$$\sigma_{core} = \frac{P}{\frac{\pi}{4}d_c^2}$$

$$\sigma_{shank} \neq \sigma_{core}$$

Method-I

making a Bolt of uniform strength

Removal of material

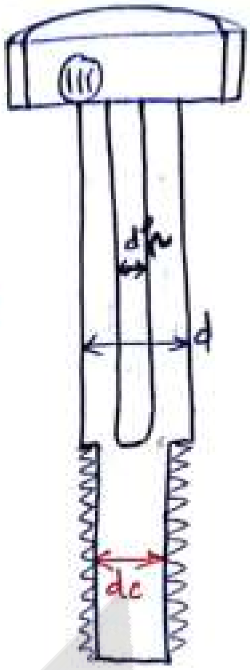



uniform strength bolt

$$\sigma_{shank} = \sigma_{core} = \frac{P}{\frac{\pi}{4}d_c^2}$$

Drilling hole in shank
Method-2

uniform strength Bolt




$$\frac{\pi}{4}(d^2 - d_h^2) = \frac{\pi}{4} d_c^2$$

$$d^2 - d_h^2 = (.84d)^2$$

$$\boxed{d_h = .542d}$$

$$\boxed{\sigma_{shank} = \sigma_{core}}$$

$$\sigma_{impact} = I.F. \ \sigma_{static}$$

$$I.F. = 1 + \sqrt{1 + \frac{2hAE}{WL}}$$

$\sigma_{imp} \downarrow \rightarrow I.F. \downarrow \Rightarrow L\uparrow, E\downarrow, h\downarrow$

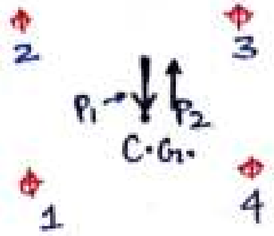
$SE \uparrow \rightarrow$ can bear more Impact/Fatigue.

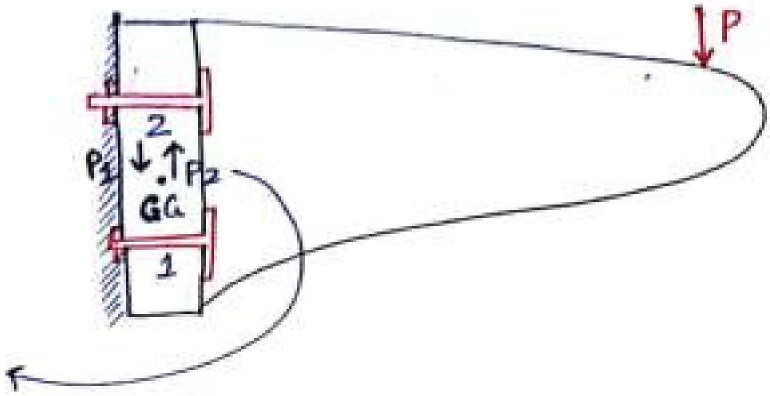
$$SE \uparrow = \frac{\sigma^2}{2E} \uparrow (\text{more})$$

uniform strength bolt $\Rightarrow$ $SE \uparrow\uparrow$ as compared to normal bolt.

→ (a) Uniform strength bolt can bear more impact and fatigue

## * Design of Bolted Joint under eccentric loading :-

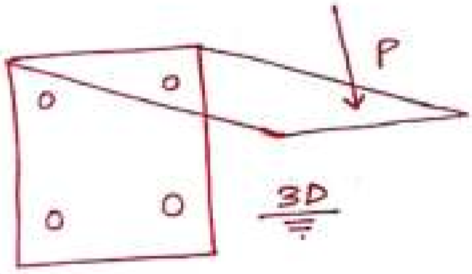


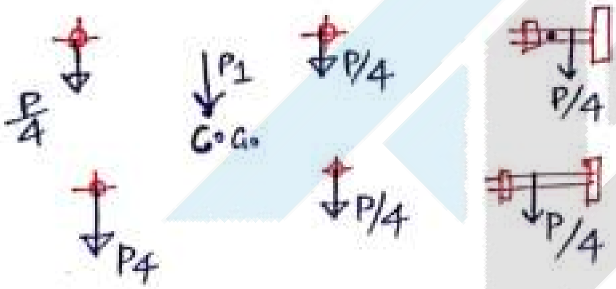




### Effect of $P_1$

⇓

Primary shear force



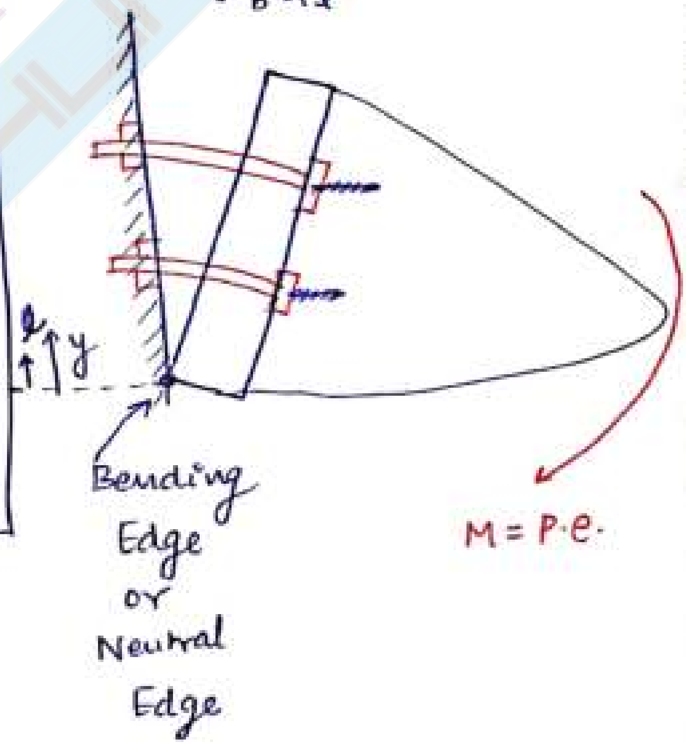




### Effect of $P_2$ and P

⇒ Bending couple

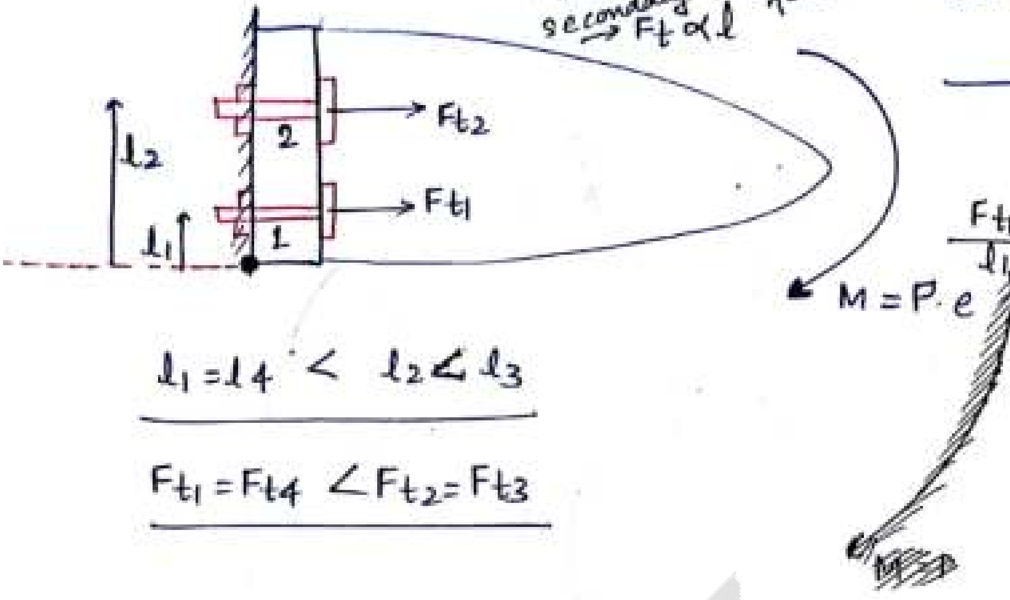
$\sigma_b \propto y$

$\sigma_b \propto l$



$\tau_b \propto l$

secondary tensile force $\rightarrow F_t \propto l$




$$F_{t1} l_1 + F_{t2} l_2 + F_{t3} l_3 + F_{t4} l_4 = P \cdot e$$

$$F_t \propto l$$

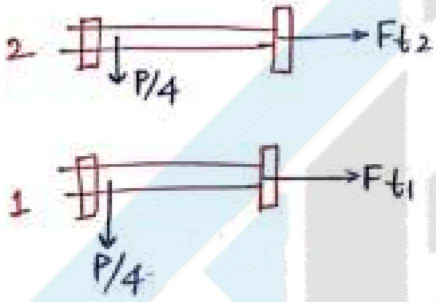
$$\frac{F_{t1}}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + l_4^2\right] = P \cdot e$$

$F_{t1}, F_{t2}, F_{t3}, F_{t4}$ are known

$$l_1 = l_4 < l_2 < l_3$$

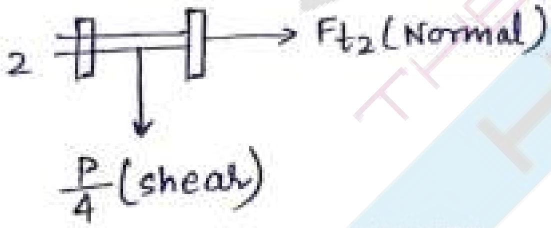
$$F_{t1} = F_{t4} < F_{t2} = F_{t3}$$

Combined effect




critical Bolt $\Rightarrow$ 2, 3

Hence bolt is in combined stress condn.




MSST $\sqrt{\sigma^2 + 4\tau^2} \le \frac{S_{yt}}{N}$

MDET $\sqrt{\sigma^2 + 3\tau^2} \le \frac{S_{yt}}{N}$

---

MSST
$$\sqrt{\left(\frac{F_{t2}}{\pi/4\, d_c^2}\right)^2 + 4\left(\frac{P/4}{\pi/4\, d_c^2}\right)^2} \le \frac{S_{yt}}{n}$$

$d_c$ = known

$d = d_c/.84$

$\boxed{d = \text{known}}$

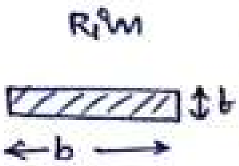
NEW CHAPTER

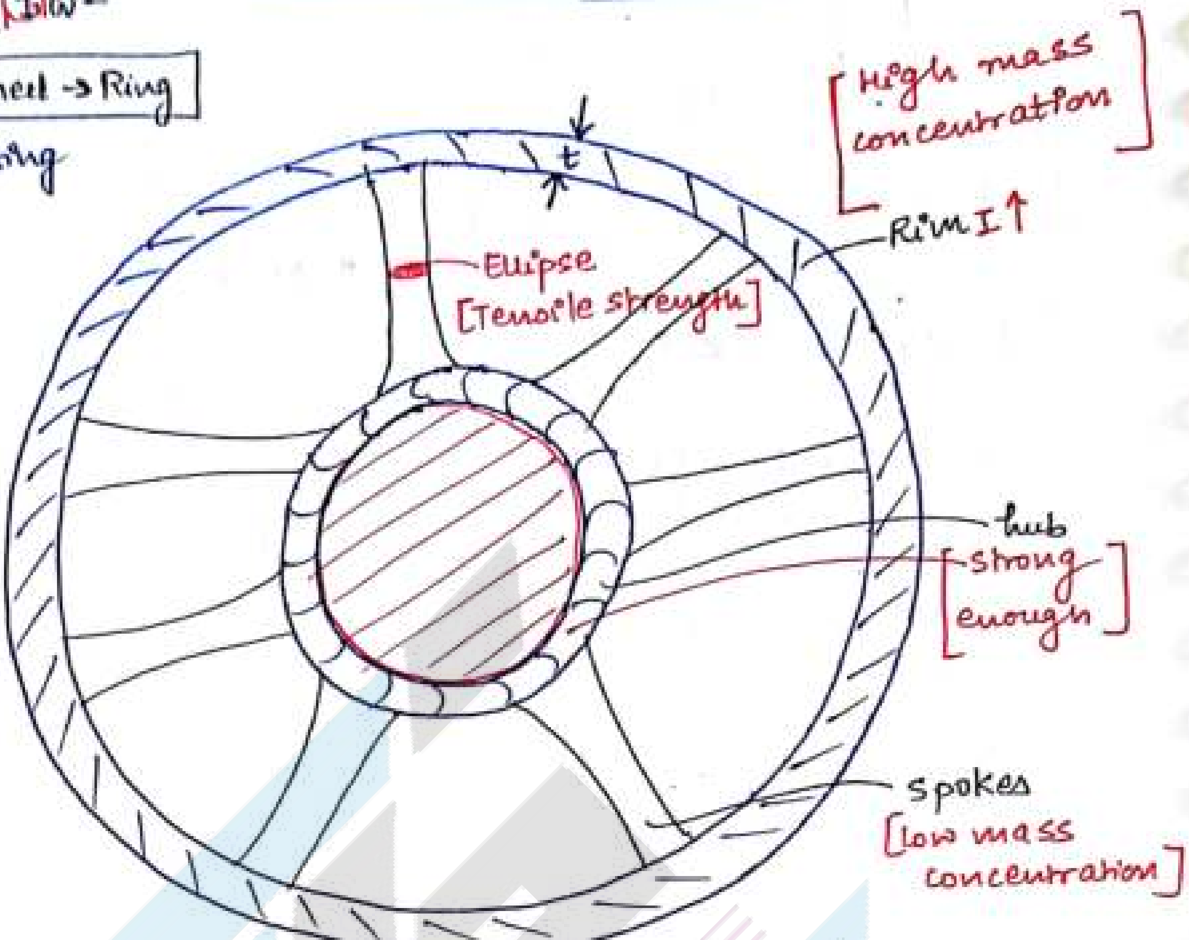
# Flywheel [Energy Bank]

$E\uparrow = \frac{1}{2} I\uparrow \omega^2$

$\boxed{\text{Best Flywheel} \rightarrow \text{Ring}}$

fly wheel → casting
fly wheel → CI




Solid one Piece flywheel
(Rimmed flywheel)

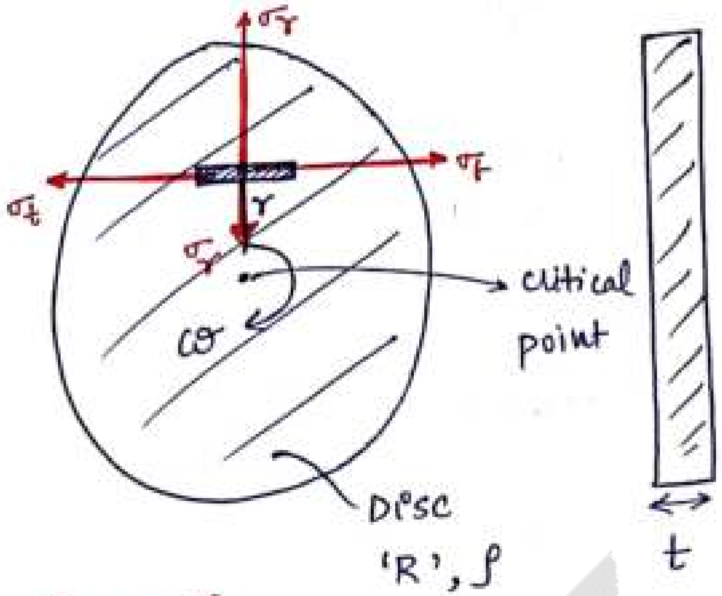
Problem
Due to Different cooling Rates

Thermal stress or cooling stress

⇓ soln.

split flywheel

## * SOLID DISC TYPE FLYWHEEL →




$\mu_{CI} = .27$ to $.3$

$I = \frac{mR^2}{2}$

$m = \rho.\pi.R^2.t$

$E = \frac{1}{2} I\omega^2$

$V = R\omega$

There are 2 types of principal stresses are induced at any generalized Radius 'r'

$\sigma_r \rightarrow$ Radial stress

$\sigma_t \rightarrow$ tangential stress

$$\sigma_t = \rho V^2 \left(\frac{\mu+3}{8}\right)\left[1 - \left(\frac{3\mu+1}{\mu+3}\right)\left(\frac{r}{R}\right)^2\right]$$

$$\sigma_r = \rho V^2 \left(\frac{\mu+3}{8}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

when $r=0 \Rightarrow \sigma_t = \sigma_r = \sigma_{max}$

$$\boxed{\sigma_{max} = \frac{\rho V^2(\mu+3)}{8}}$$ ** याद करना है

safe condn.

$\sigma_{max} \leq \sigma_{pel}$

$$\frac{\rho v^2(\mu+3)}{8} \le \sigma_{pl}$$

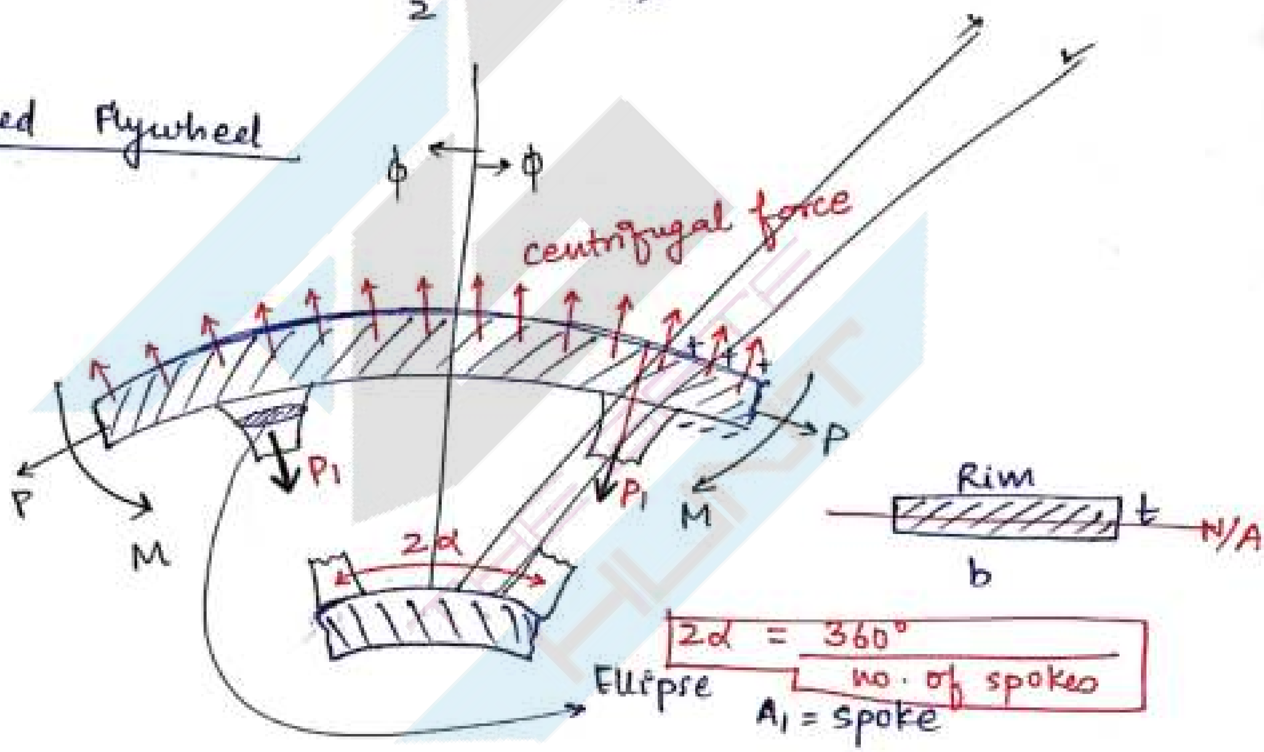
$$v \le \sqrt{\frac{8\sigma_{pl}}{\rho(\mu+3)}}$$

$$\boxed{V_{max} = \sqrt{\frac{8\sigma_{pl}}{\rho(\mu+3)}}}$$

$$\omega_{max} = \frac{V_{max}}{R}$$

$$E_{max} = \frac{1}{2} I \omega_{max}^2$$

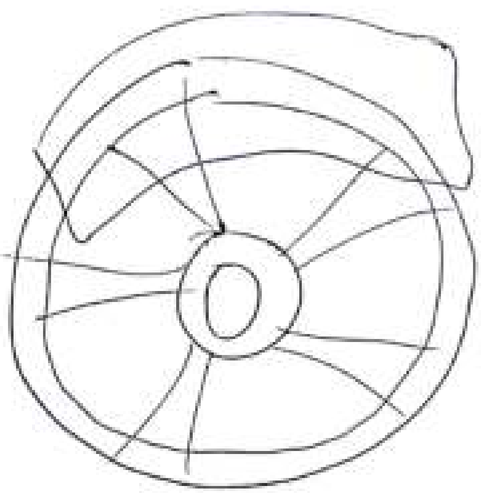
## * Rimmed Flywheel

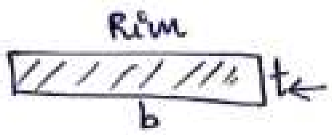



$$\boxed{2\alpha = \frac{360^\circ}{\text{no. of spokes}}}$$

$A_1$ = spoke

$A_1 = \pi(\text{semiminor})(\text{semimajor})$






$$\boxed{A = bt}$$

## Spokes Design (Tensile)

$$\sigma_{ind} = \frac{P_1}{A_1}$$

safe condn.

$$\sigma_{ind} \le P_{per}$$

$$\frac{P_1}{A_1} \le \sigma_{pr}$$

$$\boxed{A_1 = \text{Known}}$$

## Rim Design

$$\sigma_{max} = \frac{P}{bt} + \frac{6M}{bt^2}$$

safe condno

$$\sigma_{max} \le \sigma_{pl}$$

$$\frac{P}{bt} + \frac{6M}{bt^2} \le \sigma_{per}.$$

$$P_1 = \frac{2}{3}\frac{m'V^2}{C}$$

$m' \to$ mass per unit length of Rim

$$C = 12\frac{R^2}{t^2}(x) + y + \frac{A}{A_1}$$

(kg/m) $\quad m' = \frac{m}{\pi D}$

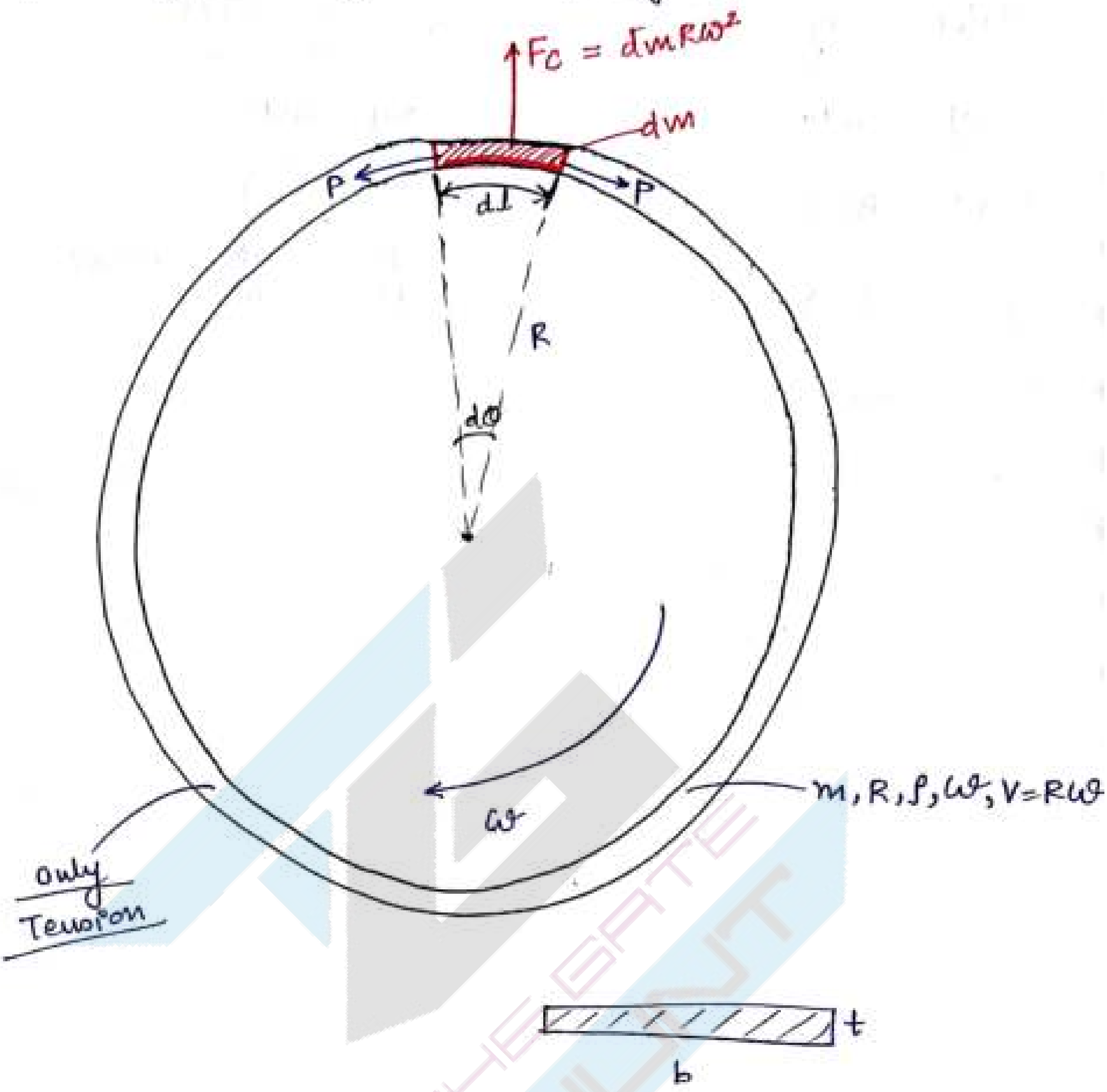
$$x = \frac{1}{2\sin^2\alpha}\left[\frac{\sin 2\alpha}{4} + \frac{\alpha}{2}\right]$$

$$y = \frac{1}{2\sin^2\alpha}\left[\frac{\sin 2\alpha}{4} + \frac{\alpha}{2}\right] - \frac{1}{2\alpha}$$

$$M_{at\ \phi} = \frac{P_1 R}{2}\left[\frac{\cos\phi}{\sin\alpha} - \frac{1}{\alpha}\right]$$

$$P_{at\ \phi} = m'V^2 - \frac{P_1\cos\phi}{2\sin\alpha}$$

* If spoke effect is neglected → Ring Flywheel

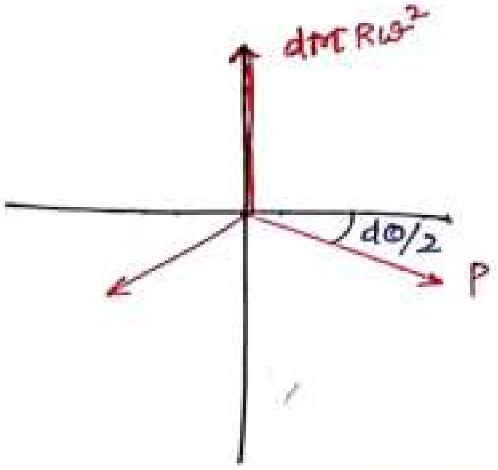



$A = bt$

$dl = R\,d\theta$

$dm = \rho \cdot dl \cdot A$

$dm = \rho(R\,d\theta) \cdot A$




$$dm\,R\omega^2 = 2P\sin\frac{d\theta}{2}$$

$d\theta \rightarrow$ Very less

$$\rho(R\,d\theta)\cdot A\cdot R\cdot\omega^2 = \cancel{2}P\cdot\frac{d\theta}{\cancel{2}}$$

$$\frac{P}{A} = \rho R^2\omega^2$$

$$\sigma_{ind} = \rho V^2$$

safe condition

$$\rho V^2 \le \sigma_{p1}$$

$$\boxed{V_{max} = \sqrt{\frac{\sigma_{p1}}{\rho}}}$$

$$\boxed{\omega_{max} = \frac{V_{max}}{R}}$$

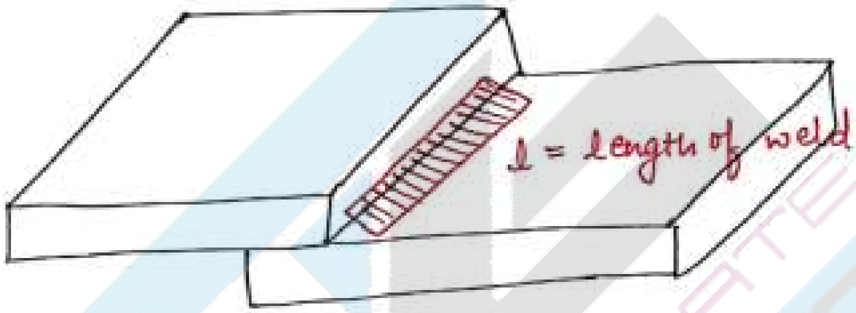
$$\boxed{E_{max} = \frac{1}{2}I\omega_{max}^2}$$

# NEW CHAPTER

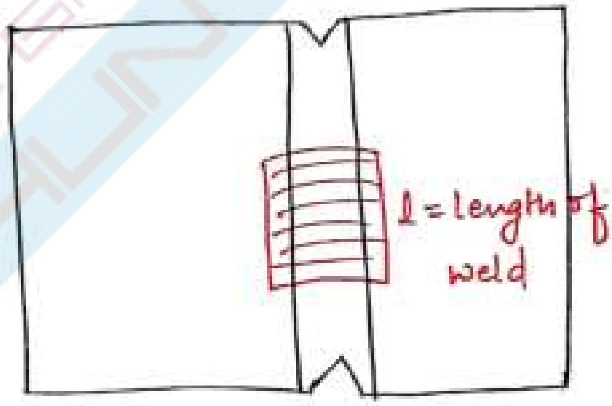
## WELDED JOINT

Fillet weld


t = size of weld/leg of weld




Buttweld

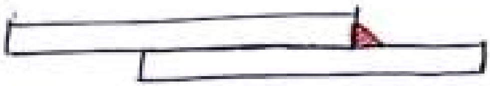
~~Flat Bolts~~ Fillet welds are weak in shear.

But Butt welds are weak in Tension.

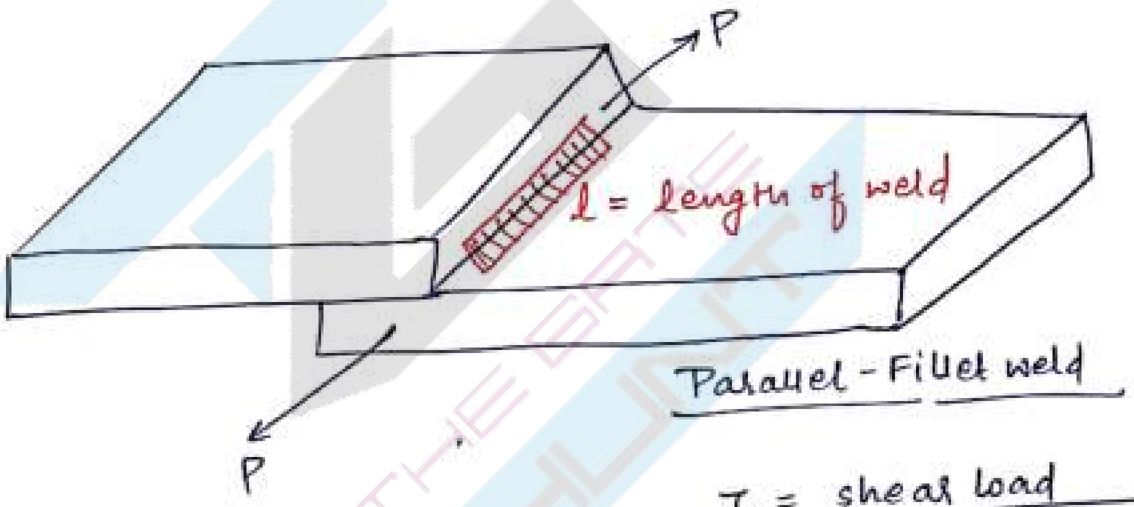
## * FILLET WELD...

① ∥ Fillet weld.

② Transverse FW.

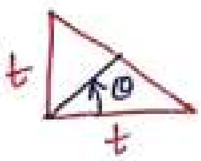


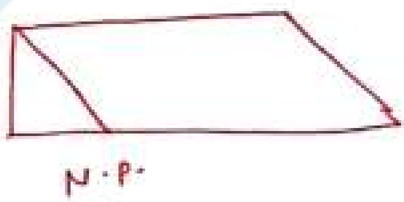
**Design of ∥ Fillet weld :-** when line of action of the load is ∥ to the length of the weld.

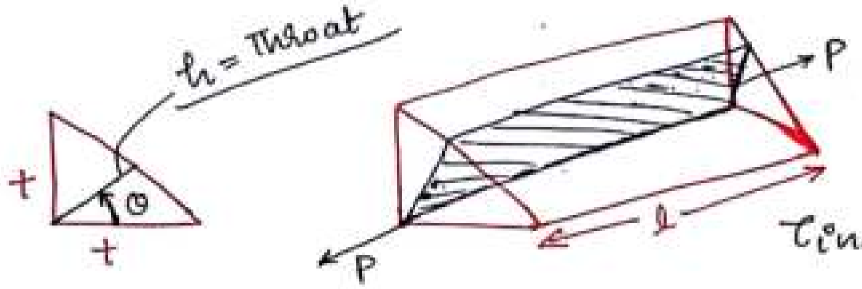

Parallel - Fillet weld

$$\tau = \frac{\text{shear load}}{\text{sheared Area}}$$



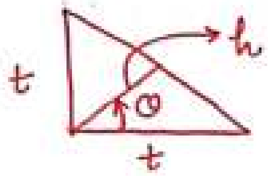

The aim of design is to determine the size of weld.




$$\tau_{ind} = \frac{P}{h\,l}$$

$$\tau_{ind} = \frac{P}{t\,l}(\sin\theta + \cos\theta).$$

$$\boxed{h = \frac{t}{\sin\theta + \cos\theta}}$$




Parallel Fillet weld ⇒ only shear stress

$\tau_{ind} \longrightarrow \tau_{max}$

$$\tau_{ind} = f(\theta)$$

$$\frac{d\tau_{ind}}{d\theta} = 0$$

$$\boxed{\begin{aligned}\theta &= 45^\circ \\ h &= .707\,t\end{aligned}} \Rightarrow \text{condition for } \tau_{max}$$

$$\tau_{max} = \frac{P}{.707\,t\,l}$$

<u>safe condition</u>

$$\tau_{max} \leq \tau_{pl}$$

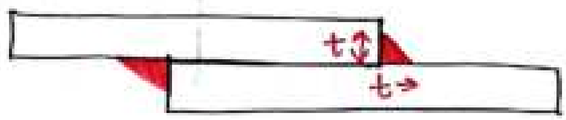
$$\frac{P}{.707\,t\,l} \leq \tau_{pl}$$

$$\boxed{P_{max} = .707\, t\, l \cdot \tau_{pr}}$$

⇓

shear strength of Parallel Fillet weld

if Double Parallel

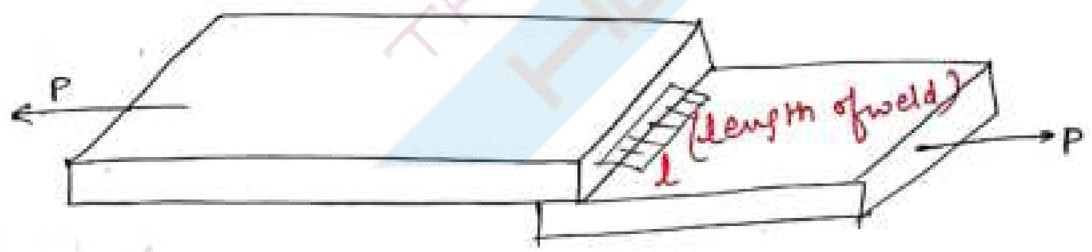


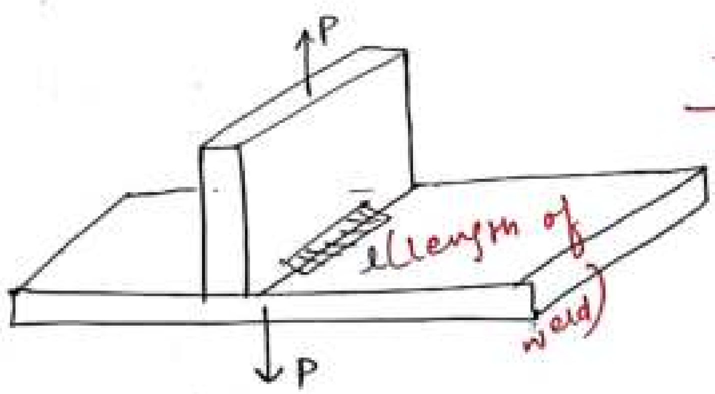

$$\tau_{max} = \frac{P}{2 \times .707\, t \cdot l}$$

$$P_{max} = 2 \times .707\, t \cdot l \cdot \tau_{pr}$$

* <u>Transverse Fillet weld</u> :-

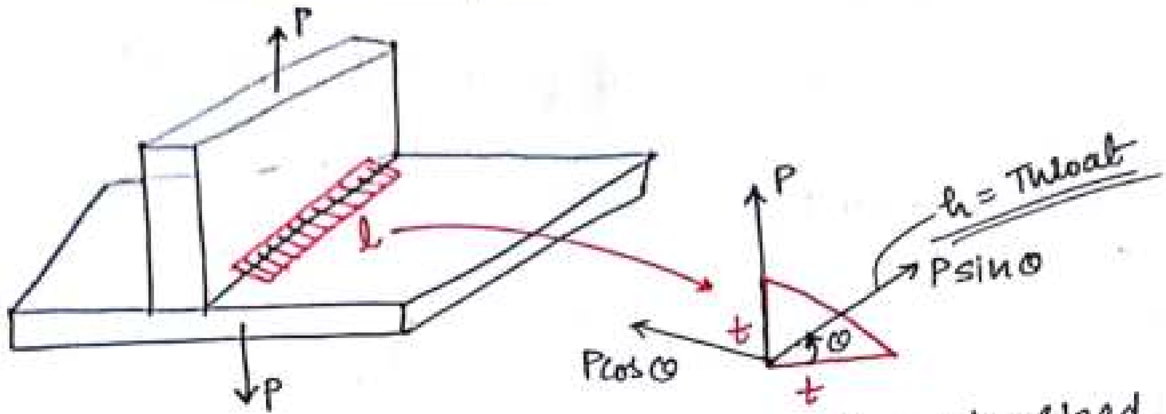

when the line of action of the load is ⊥r to the length of weld referred as Transverse weld.



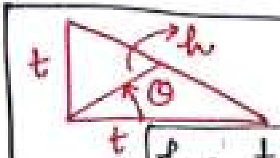




<u>Transverse Fillet weld</u>

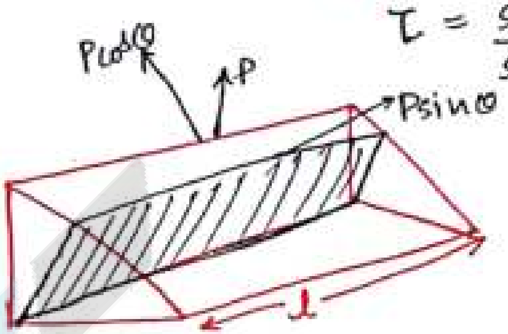
## Now Design




$\tau = \dfrac{\text{shear load}}{\text{shear area}}$




$$\boxed{h = \frac{t}{\sin\theta + \cos\theta}}$$




Transverse Fillet weld ⇒ shear + Tension

$$\tau_{ind} = \frac{P\sin\theta}{h\,l}$$

$$\tau_{ind} = \frac{P}{t\,l}(\sin\theta + \cos\theta)\sin\theta$$

$\tau_{ind} \rightarrow \tau_{max}$

$\tau_{ind} = f(\theta)$

$\dfrac{d\tau_{ind}}{d\theta} = 0$ ⇒ cond^n. for $\tau_{max}$

$$\boxed{\begin{aligned}\theta &= 67\tfrac{1}{2}^\circ \\ h &= .765t\end{aligned}}$$

$$\tau_{max} = \frac{P}{.828t \cdot l}$$

Safe cond^n

$\tau_{max} \le \tau_{per}$

$$\frac{P}{.828tl} \le \tau_{pl}$$

$$\boxed{P_{max} = .828tl \cdot \tau_{pl}}$$

⇓

shear strength of Transverse Fillet weld

if Double :

$$\tau_{max} = \frac{P}{2 \times .828tl}$$

$$P_{max} = 2 \times .828tl\,\tau_{pl}$$

कोई भी fillet weld को Parallel से Design करो

$(P_{max})_{parallel} = .707\,t \cdot l \cdot \tau_{pr}$

$(P_{max})_{Transverse} = .828\,t \cdot l \cdot \tau_{pr}$

Note :- hence, parallel is weaker than Transverse, so always design any flat belt by assuming parallel flat weld only

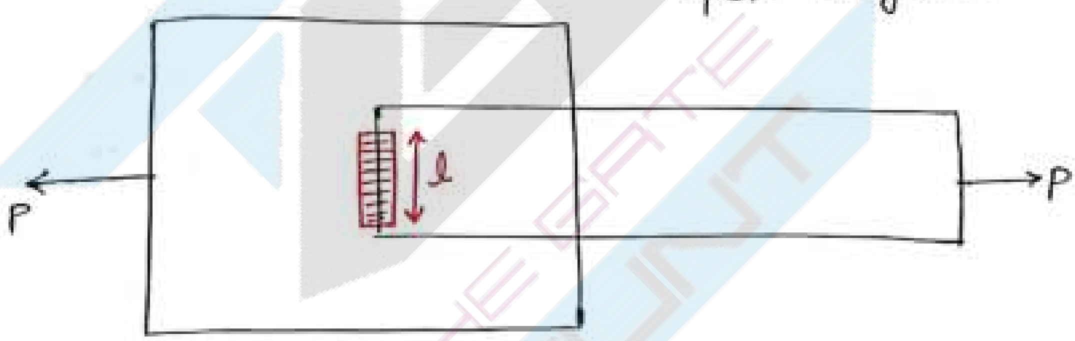
$$\boxed{\begin{array}{l} \text{min}^m\text{. thickness} \approx .707\,t \ (\text{Throat} =) \\ \text{min}^m\text{. Area} = .707\,t\,l \end{array}}$$

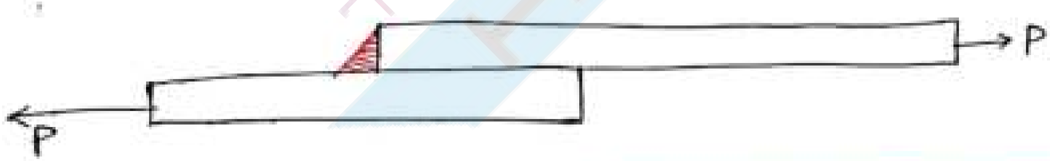
for any Fillet weld Parallel / Transverse

## * Fillet Weld Under Tension :-

$\sigma_{per}$ = given

$\tau_{per}$ = Not given




$$\boxed{\sigma_{max} = \frac{P}{\underbrace{.707\,t\,l}_{\text{James Bond}}}}$$

Safe condn.

$\sigma_{max} \le \sigma_{pl}$

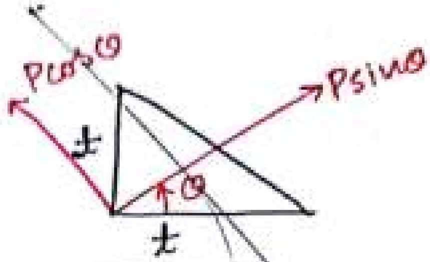
$\dfrac{P}{.707tl} \le \sigma_{pl}$

$$\boxed{P_{max} = .707\,tl\cdot\sigma_{pl}}$$

$\Downarrow$

Tensile strength of fillet weld.

Note नहीं करना




$\sigma_{ind} = \dfrac{P\cos\theta}{tl}(\sin\theta + \cos\theta)$

$\dfrac{\sigma_{ind}}{d\theta} = 0$

$\theta = 22\tfrac{1}{2}°$

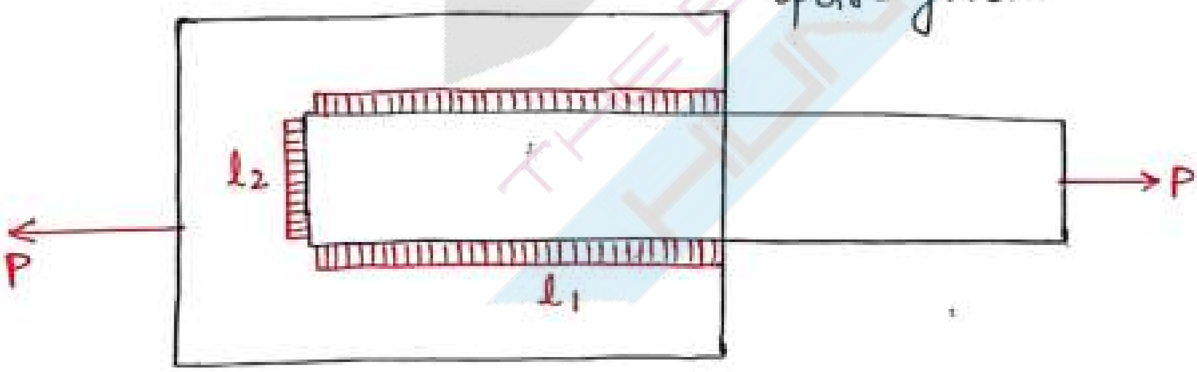
$$\boxed{(\sigma_{ind})_{max} = \frac{P}{.828tl}}$$

$\dfrac{P}{.707tl} > \dfrac{P}{.828tl}$

Q.

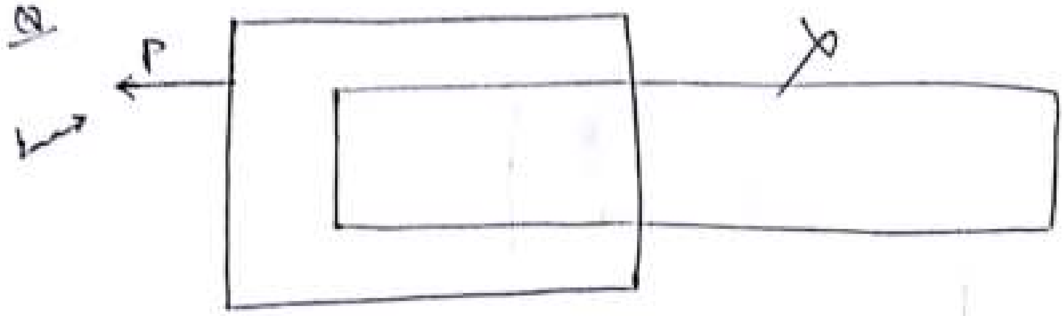
$\sigma_{pel.}$ = given

$\tau_{pel.}$ = given



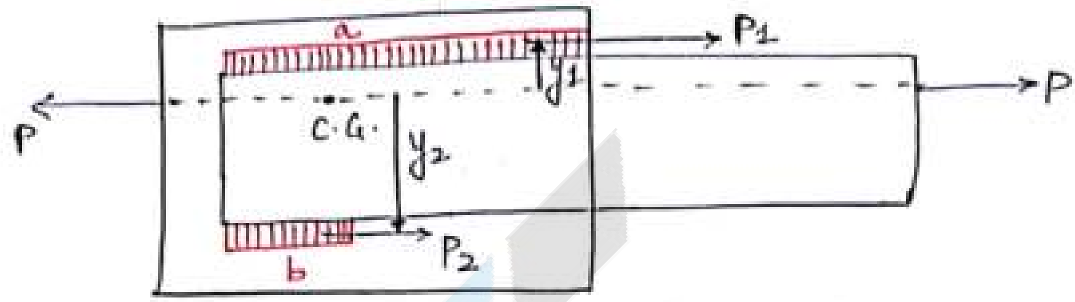

$P_{max} = 2\times .707\,tl_1\cdot\tau_{pl} + .707\,tl_2\,\sigma_{pl}.$

Q




$t$ = given

$a, b = ?$




$$P_1 + P_2 = P$$

$$.707\,t \cdot a \cdot \tau_{pl} + .707\,t \cdot b \cdot \tau_{pr} = P \text{ — (i)}$$

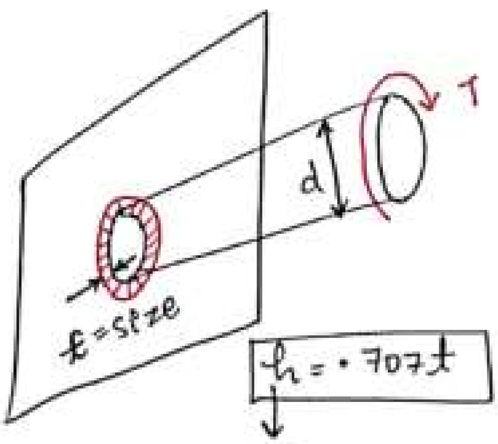
moment about G.G

$$P_1 y_1 = P_2 y_2$$

$$a y_1 = b y_2 \text{ — (ii)}$$

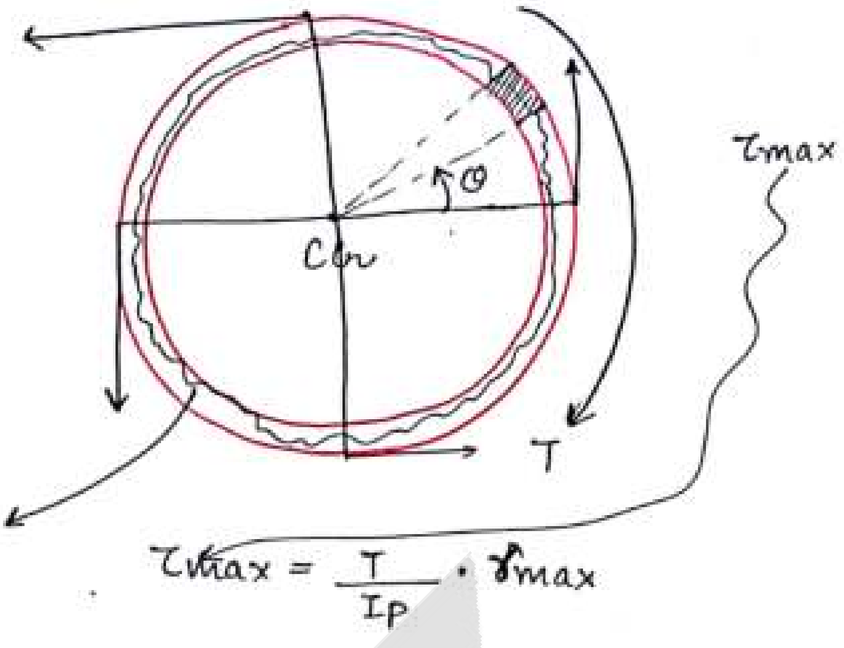
a, b are known

## * Fillet weld under Twisting :-



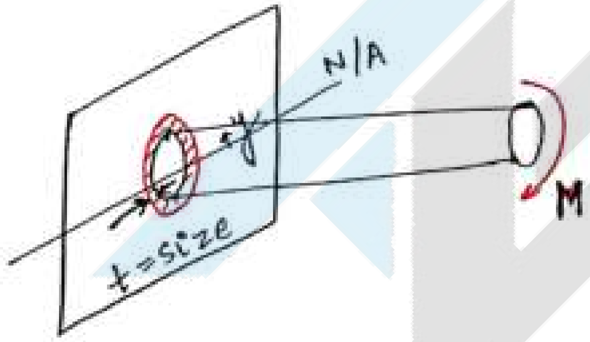

$$\tau_{max} = \frac{2.83\,T}{\pi d^2 t}$$

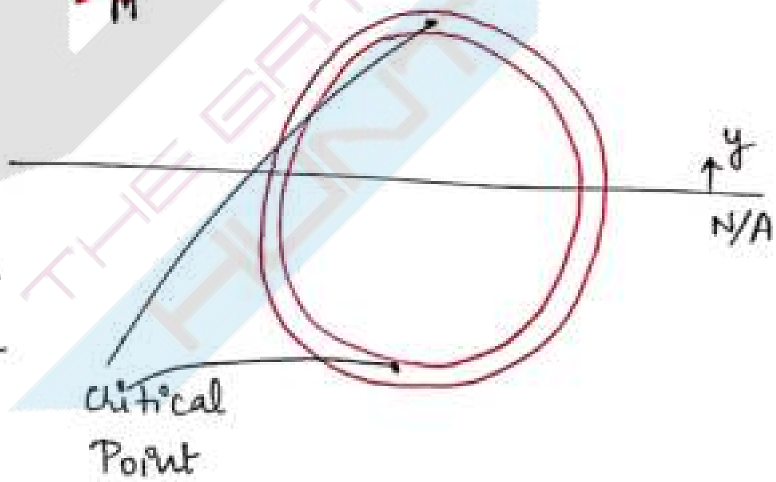
$\boxed{h = .707t}$ → Throat




$$\tau_{max} = \frac{T}{I_p} \cdot r_{max}$$

* Flat weld under Bending →

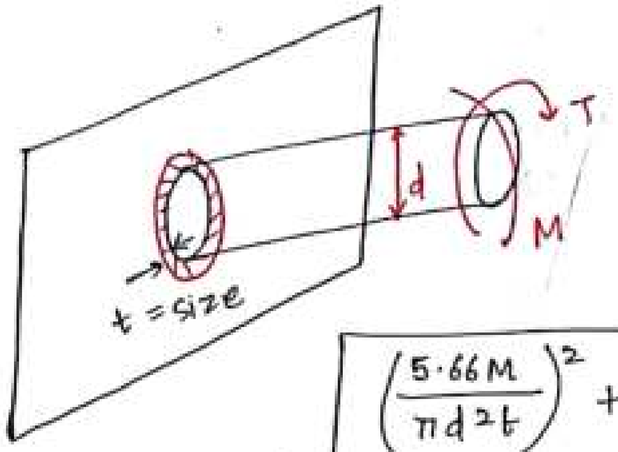



$$\sigma_{b\,max} = \frac{5.66\,M}{\pi d^2 t}$$

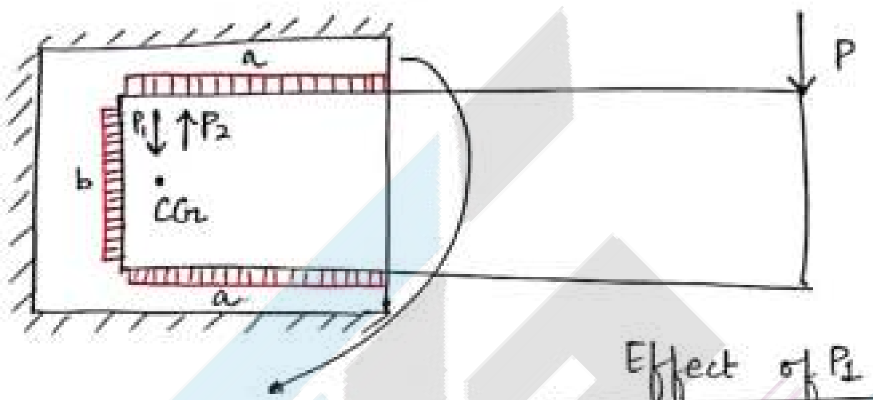
t = size

$\tau \propto \gamma$

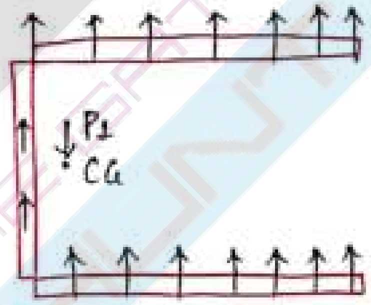



$$\sqrt{\left(\frac{5.66M}{\pi d^2 t}\right)^2 + 4\left(\frac{2.83T}{\pi d^2 t}\right)^2} \le \frac{S_{yt}}{N}$$

Q.




Effect of $P_1$

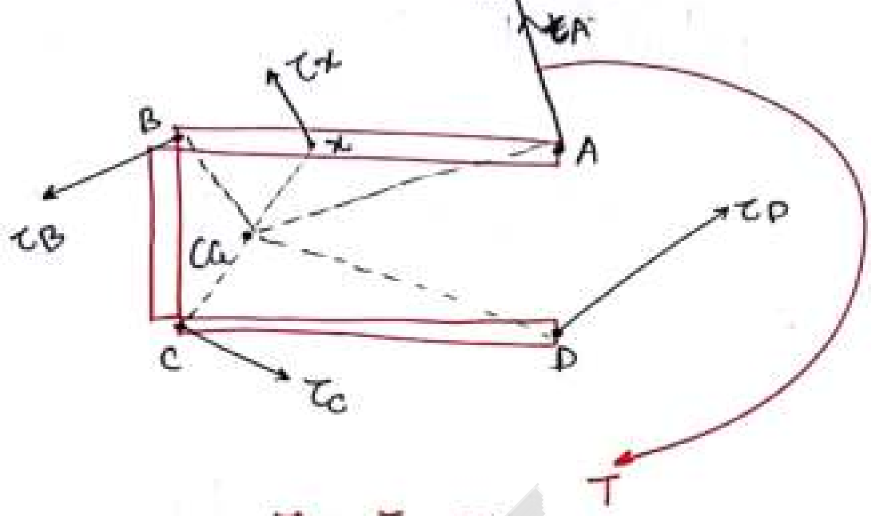



$\tau$

$$\tau_{direct} = \frac{P}{.707t(2a+b)}$$

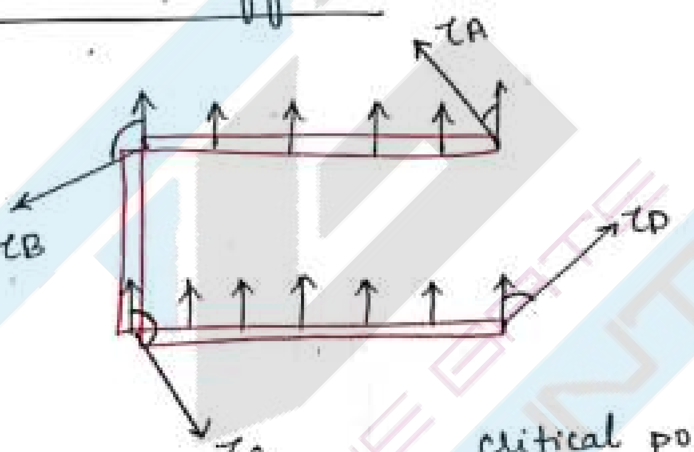
Effect of $P_2$ and P → Twisting couple

$\tau_{torsional} \propto r$

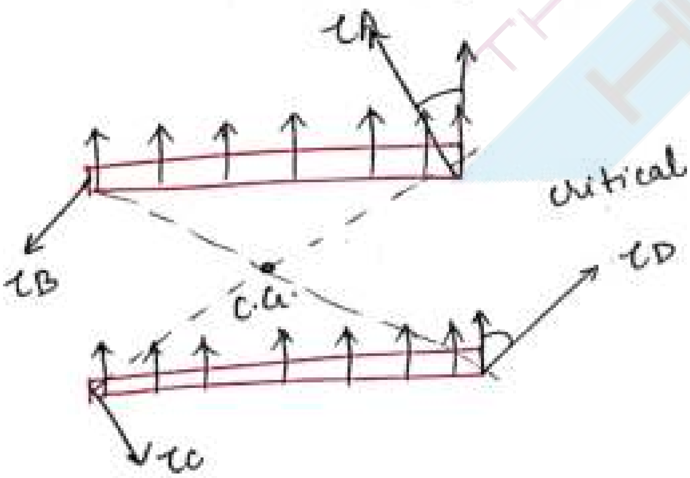



$$\tau = \frac{T}{I_p} \cdot r$$

## Combined Effect



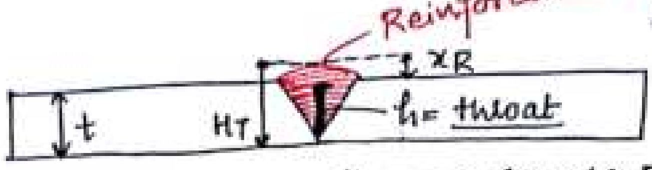

critical point = A, D




critical point A, D

## Butt Joint

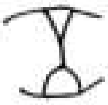
$\boxed{h = t}$

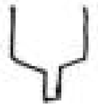



$t < 6\,mm \longleftarrow \| \longrightarrow$ square Butt weld ⊓

$20\,mm \geq t \geq 6\,mm$ $V \longrightarrow$ vee- buttweld

$t > 20\,mm$ $X \longrightarrow$ Double Vee- butt weld



 $\longrightarrow$ U- butt weld

$\boxed{h = H_T - x_R}$

The height of Reinforcement will not be considered in the design because Reinforcement will be grind out after the welding is completed.

$$(\sigma_{ind})_{max} = \frac{P}{hl}$$

safe condn.

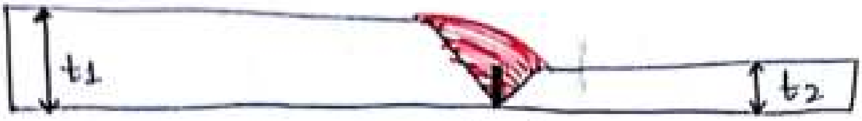
$$\sigma_{max} \leq \sigma_{pr}$$

$$\frac{P}{hl} \leq \sigma_{per}$$

$$\boxed{P_{max} = hl \cdot \sigma_{pr}}$$

$\Downarrow$

tensile strength of Buttweld




$$h = t_2$$

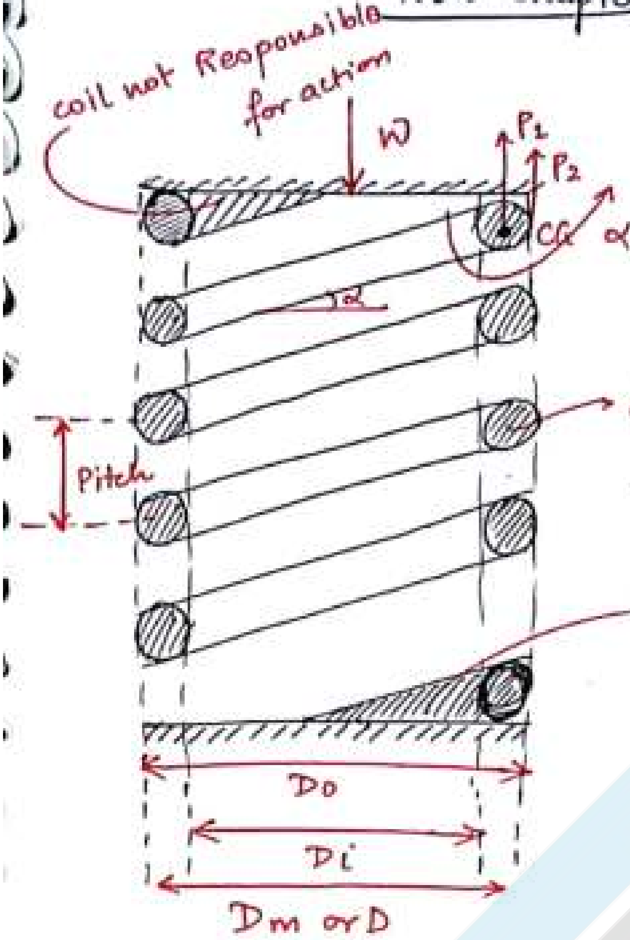

$$h = t - y$$


$$h = t - d$$

weld strength $\propto$ skillness of labous

NEW Chapter

# SPRING




$\alpha \rightarrow$ helix∠

$n$ = no. of Active coils

$D_o$ = outer dia of coil

$D_i$ = inner — " —

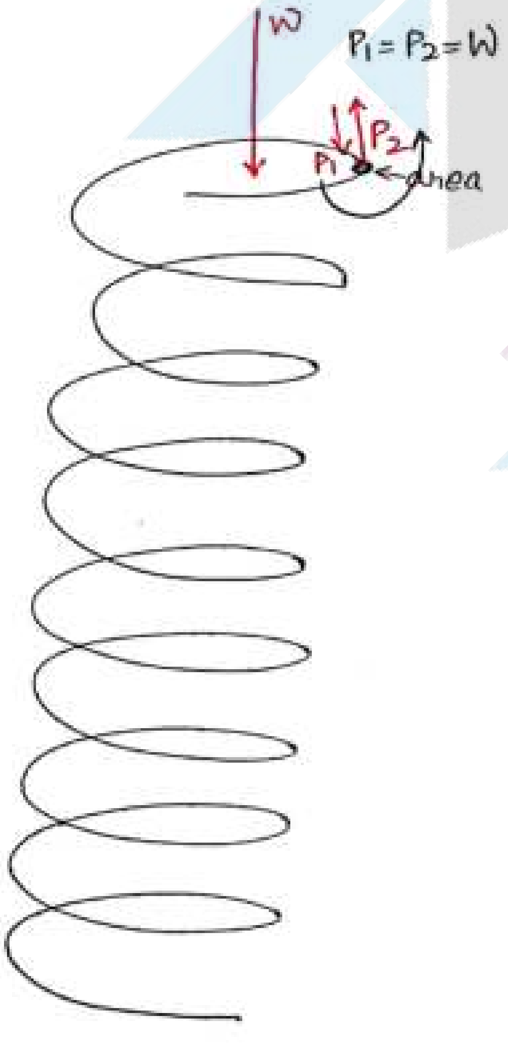
$D_m$ or $D$ = mean — " —

$D = D_o - d = D_i + d$

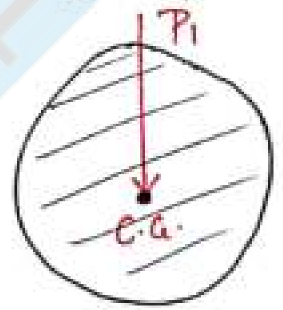
C = spring index

$$\boxed{C = \frac{D}{d}}$$

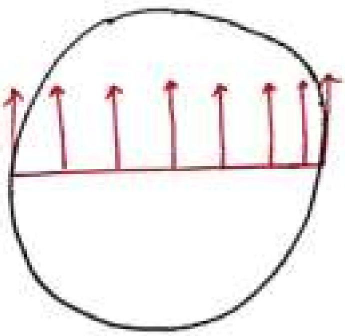
$$\boxed{4 \le C \le 12}$$




## Effect of $P_1$

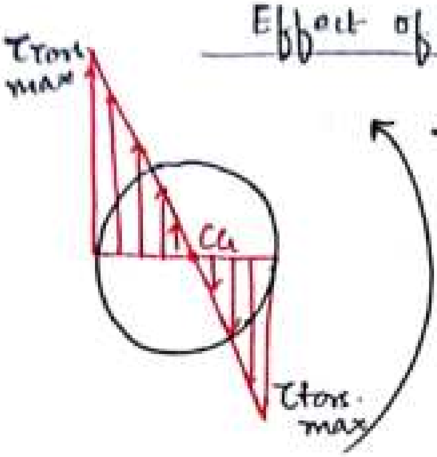



⇒ direct shear stress



dir. sh. stress

$$\tau_{dir\ av} = \frac{W}{\frac{\pi}{4} d^2} = \frac{4W}{\pi d^2}$$



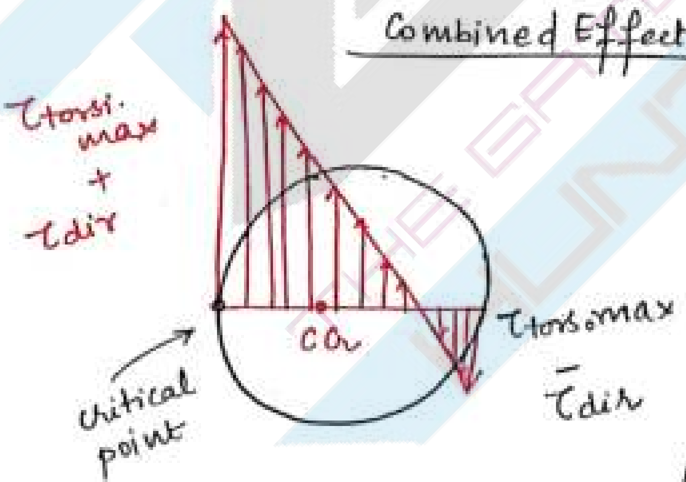

## Effect of B and P

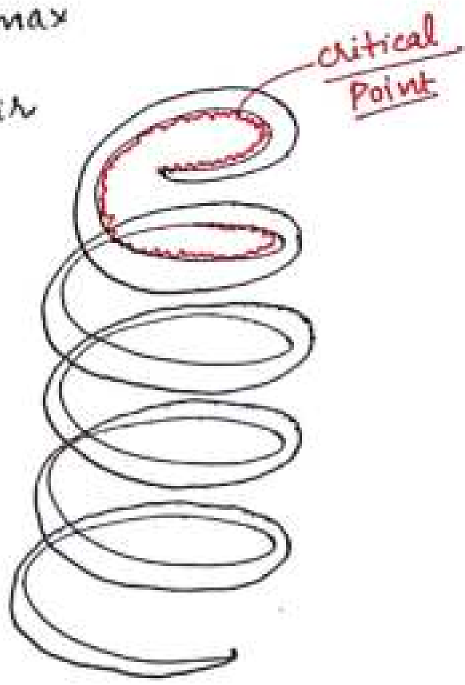
$$T = \frac{WD}{2}$$

$$\tau_{max\ torsional} = \frac{16T}{\pi d^3} = \frac{16\left(\frac{WD}{2}\right)}{\pi d^3}$$

$$\tau_{tors\ max} = \frac{8WD}{\pi d^3}$$

## Combined Effect




$$\tau_{max} = \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

$$\tau_{max} = \frac{8WD}{\pi d^3}\left[1 + \frac{.5}{c}\right]$$

Conclusion → ① hence, spring wire is subjected to only shear. Torsional and direct shear both under spring subjected to tension and compression.

② Inner fibre of the spring wire is subjected to maximum stress.

$$\tau_{max} = \underbrace{\frac{8WD}{\pi d^3}}_{\text{Twisting}} \underbrace{\left[1 + \frac{.5}{c}\right]}_{\text{direct shear}}$$

$K_{sh}$ = Direct shear stress factor

$$\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_{sh}$$

$K_c$ = curvature Effect factor

$$\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_{sh} \cdot K_c$$

$K_{sh} \cdot K_c = K_w$ ↳ wahl's factor

$$K_w = \frac{4c-1}{4c-4} + \frac{.615}{c}$$

$$\tau_{max} = \frac{8WD}{\pi d^3} \cdot K_w$$

hence, $K_w$ factor include direct shear and curvature both.

safe condn.

$$\tau_{max} \le \tau_{ps}$$

$$\frac{8WD}{\pi d^3} K_w \le \tau_{pl}$$

$$\boxed{W_{max} = \frac{\pi d^3}{8DK_w} \cdot \tau_{pr}}$$

compr./Tensile strength of spring

## * Expression for Deflection and stiffness :-

$$SE = \frac{1}{2} T\theta = \frac{1}{2} \frac{T^2 L}{G \cdot J} = \frac{1}{2}\left(\frac{WD}{2}\right)^2 \frac{\pi D n}{G \cdot \frac{\pi}{32} \cdot d^4}$$

$$\boxed{U = \frac{4W^2 D^3 n}{G d^4}}$$

Catigliaro Theorem →

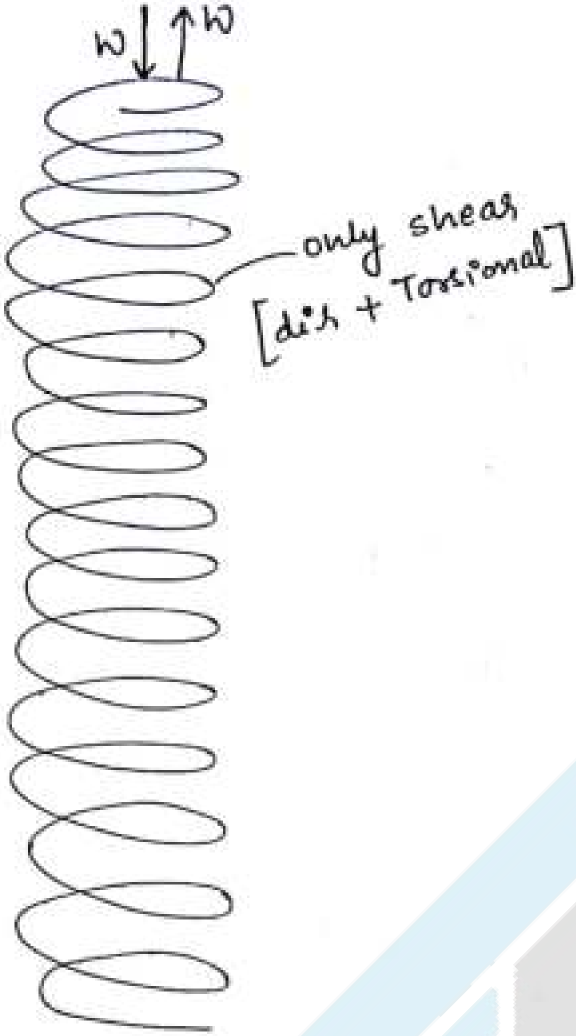
$$\delta = \frac{\partial U}{\partial W} = \frac{8WD^3 n}{G d^4}$$

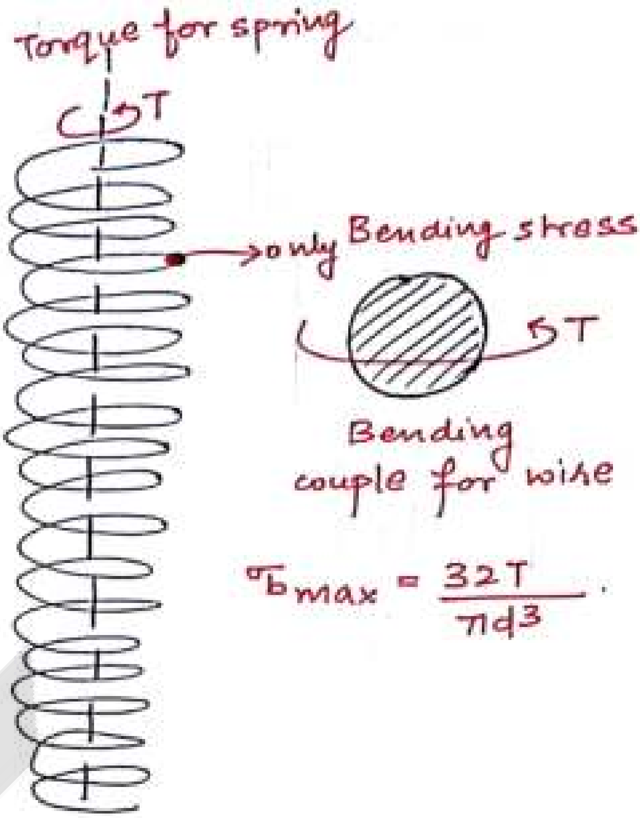
$$\boxed{\delta = \frac{8WD^3 n}{G d^4}}$$

$$K = \frac{W}{\delta}$$

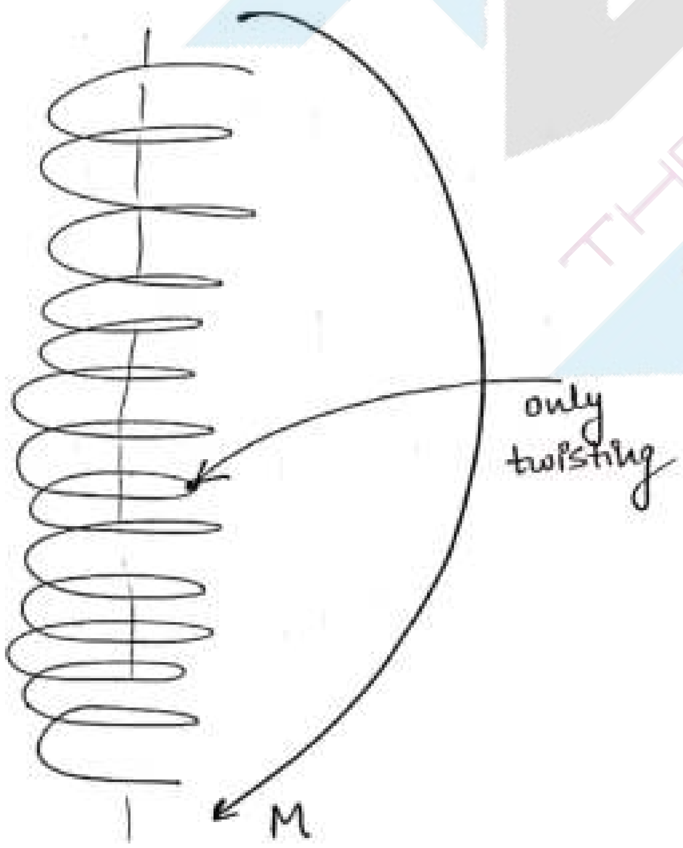
$$\boxed{K = \frac{G d^4}{8 D^3 n}}$$


$$\boxed{K \propto 1/n}$$




$$\sigma_{b\,max} = \frac{32T}{\pi d^3}.$$

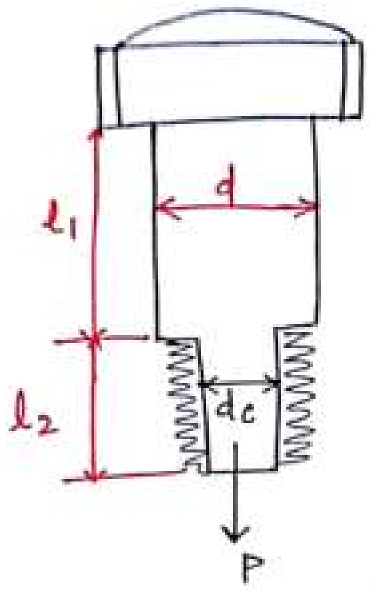




Twisting couple

$$\boxed{\tau_{max} = \frac{16M}{\pi d^3}}$$

## "Bolt" as a spring




$$K_{shank} = \frac{\pi}{4}\frac{d^2 . E}{l_1}$$

$$K_{core} = \frac{\pi}{4}\frac{d_c^2 . E}{l_2}$$

$\Rightarrow$

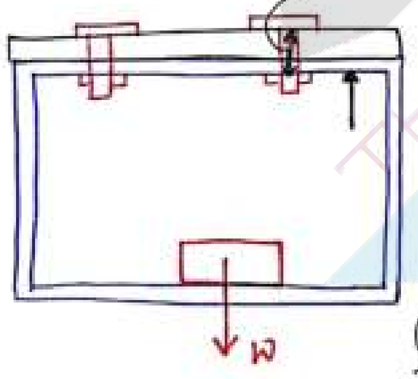
$$\frac{1}{K_{eq}} = \frac{1}{K_{shank}} + \frac{1}{K_{core}}$$

$$K = \frac{P}{\delta} = \frac{P}{\frac{PL}{AE}}$$

$$K = \frac{AE}{L}$$

## * Pre- Tensioning of the Bolt.

$F_{pretension} = F_{clamped}$




Case(1)

$W > F_{pretension}$

(i) $W_{Net\ Bolt} = W - F_{pretension}$

Strength ↑

(ii) $SE \uparrow = \frac{\sigma^2}{2E} \uparrow$ ( )

more Impact and Fatigue.

(iii) Leak Proof.

Case(2)

$W < F_{pretension}$

$$\boxed{W_{Net\ Bolt} = F_{pretension}}$$

Load loss

① Vibrⁿ.

② Creep.

③ yielding in thread.